

Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Summer 2023 (S61)

Solutions to Assignment #1

Plotting with SageMath

Due just before midnight on Friday, 12 May.

Before tackling this assignment, please read *Getting Started with sage.trentu.ca*, if you haven't already, as well as take a peek at the file `1110H-1ab-20230503.pdf`, both of which you can find in the Labs folder in the Course Content section on Blackboard. Skimming and later referring to as necessary to Sections 1.4, 3.3, 3.4, and 3.6 of Gregory Bard's book *Sage for Undergraduates*, which you can find in the SageMath folder in the Course Content section on Blackboard, is probably a good idea. (There are other possibly useful resources in that folder, too.) Should you wish to use another general purpose mathematics application, such as Maple or Mathematica, you may, but you're on your own for learning to use it and getting help.

In each of questions 1–4, use SageMath, or a comparable program, to plot the required graphs.

- Plot the following graphs in Cartesian coordinates. [2 = 4 × 0.5 each]
 - $y = x^3 - x$ for $-2 \leq x \leq 2$.
 - $y = 2^x$ for $-1 \leq x \leq 1$.
 - $y = \sin(\pi x)$ for $0 \leq x \leq 4$.
 - $y = \sin(\pi x^2)$ for $0 \leq x \leq 4$.
- Plot the following implicitly defined curves. [2 = 4 × 0.5 each]
 - $(x - 1)^2 + (y - 2)^2 = 4$ for $-1 \leq x \leq 3$ and $0 \leq y \leq 4$.
 - $x + y = x^2 + y^2$ for $-0.5 \leq x \leq 1.5$ and $-0.5 \leq y \leq 1.5$.
 - $4x^2 + 9y^2 = 36$ for $-3 \leq x \leq 3$ and $-2 \leq y \leq 2$.
 - $(x^2 + y^2)^2 + 4x(x^2 + y^2) - 4y^2 = 0$ for $-5 \leq x \leq 1$ and $-3 \leq y \leq 3$.
- Plot the following parametric curves. [2 = 4 × 0.5 each]
 - $x = 3 \cos(t)$ and $y = 2 \sin(t)$ for $0 \leq t \leq 2\pi$.
 - $x = t \cos(t)$ and $y = t \sin(t)$ for $0 \leq t \leq 2\pi$.
 - $x = t^3$ and $y = t^9 - t^3$ for $-1 \leq t \leq 1\pi$.
 - $x = 2(1 - \cos(t)) \cos(t)$ and $y = 2(1 - \cos(t)) \sin(t)$ for $0 \leq t \leq 2\pi$.
- Plot the following curves in polar coordinates. [2 = 4 × 0.5 each]
 - $r = 2^\theta$ for $-\pi \leq \theta \leq \pi$.
 - $r = 2 + 3 \sin(\theta)$ for $-\pi \leq \theta \leq \pi$.
 - $r = 4 \cos(\theta)$ for $0 \leq \theta \leq \pi$.
 - $r = 4 \cos(2\theta)$ for $0 \leq \theta \leq \pi$.
- What is the equation in x and y that implicitly defines the curve given in polar coordinates in 4c above? [2]

SOLUTION. The given curve is described by $r = 4 \cos(\theta)$ in polar coordinates. Recall from class that if a point in the plane has polar coordinates (r, θ) and Cartesian coordinates

(x, y) , then $r^2 = x^2 + y^2$, $x = r \cos(\theta)$, and $y = r \sin(\theta)$. It follows from the first that $r = \sqrt{x^2 + y^2}$ and from the first and second that $\cos(\theta) = \frac{x}{r} = \frac{x}{\sqrt{x^2 + y^2}}$. Plugging these into the polar equation of the curve gives us an equation for the curve in Cartesian coordinates:

$$r = 4 \cos(\theta) \implies \sqrt{x^2 + y^2} = 4 \cdot \frac{x}{\sqrt{x^2 + y^2}}$$

We can make this Cartesian equation look nicer, and avoid the risk of dividing by 0, by multiplying the Cartesian equation by $\sqrt{x^2 + y^2}$ on both sides to get:

$$x^2 + y^2 = 4x$$

A small question for you: How do the limits given for θ in **4c**, $-\pi \leq \theta \leq \pi$, matter to the Cartesian equation we obtained above? \square

NOTE: For those interested in what this curve is, the plot for **4c** strongly suggests that it is the circle with radius 2 and centre $(2, 0)$. We can verify this pretty easily, now that we have the Cartesian equation for the curve in question. The standard form of the equation in Cartesian coordinates of the circle with radius 2 and centre $(2, 0)$ is $(x - 2)^2 + y^2 = 2^2$. We will expand, simplify, and rearrange this equation below.

$$\begin{aligned} (x - 2)^2 + y^2 = 2^2 &\iff x^2 - 4x + 4 + y^2 = 4 \\ &\iff x^2 - 4x + y^2 = 0 \\ &\iff x^2 + y^2 = 4x \end{aligned}$$

That is, the Cartesian equation $x^2 + y^2 = 4x$ we obtained that is equivalent to the polar curve $r = 4 \cos(\theta)$ is also equivalent to the standard form of the equation in Cartesian coordinates of the circle with radius 2 and centre $(2, 0)$. Thus the curve in question really is the circle with radius 2 and centre $(2, 0)$.

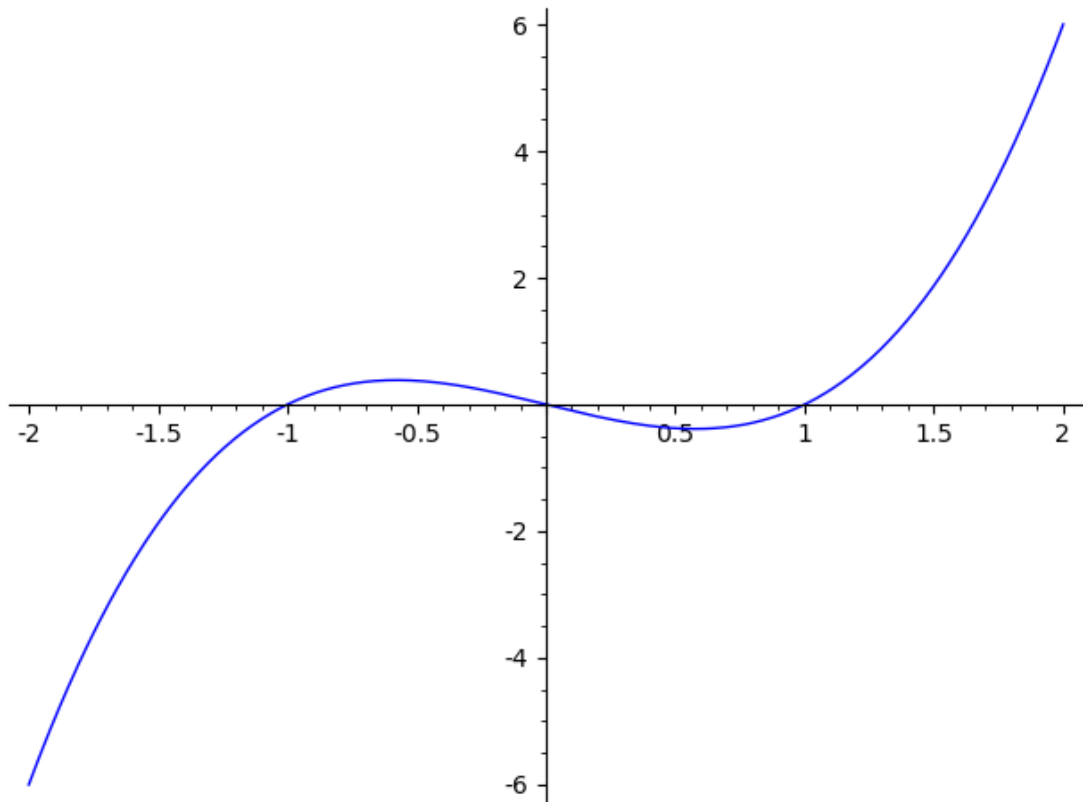
The solutions to questions 1–4 are appended below.

MATH1110H-A1-Solutions-questions-1-4

May 9, 2023

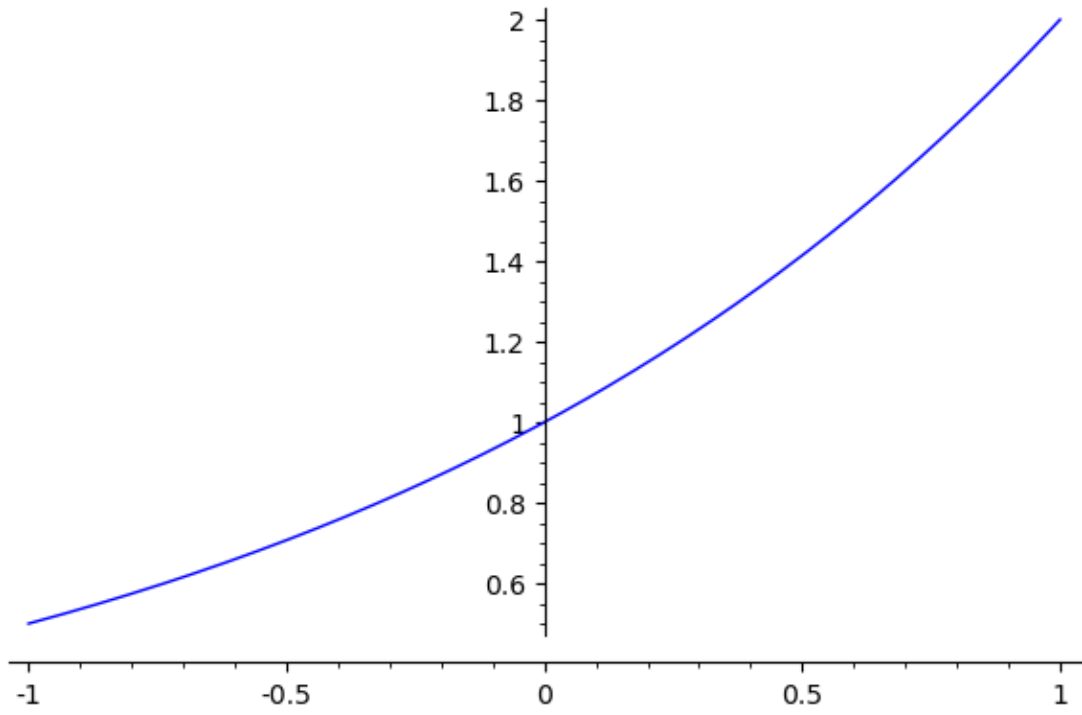
```
[1]: # MATH 1110H Summer 2023 (S61)
# Assignment #1 - Plotting with SageMath
# Solutions to Questions 1-4
#
# 1a
plot(x^3 - x, -2, 2)
```

[1]:



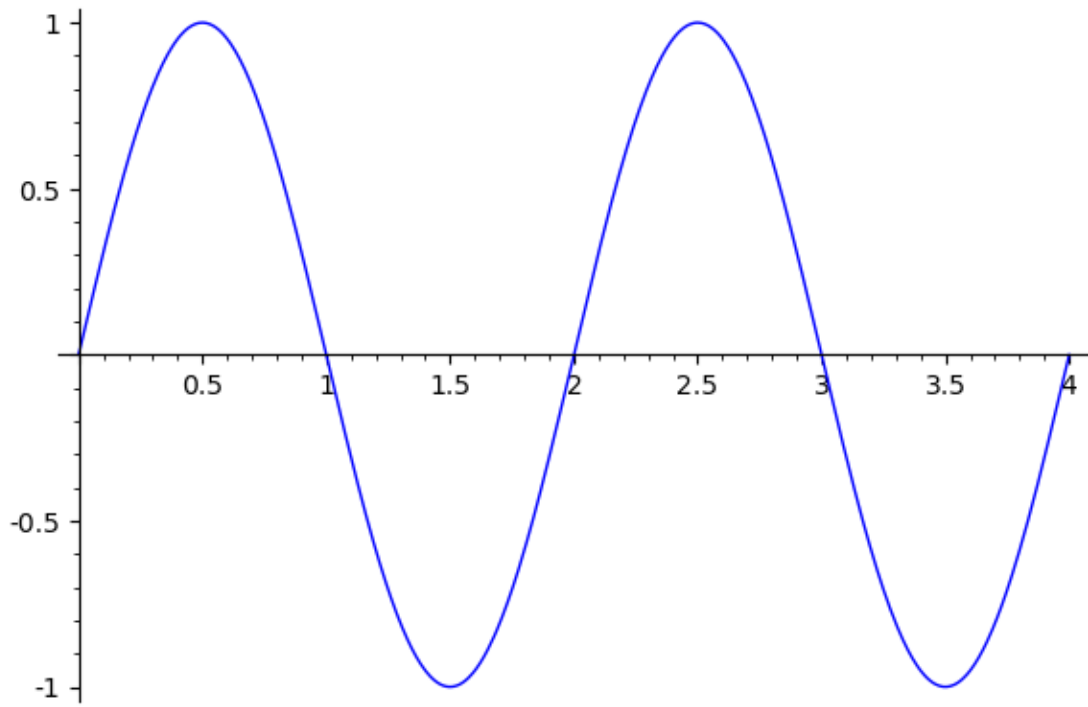
```
[2]: # 1b
plot(2^x, -1, 1)
```

[2]:



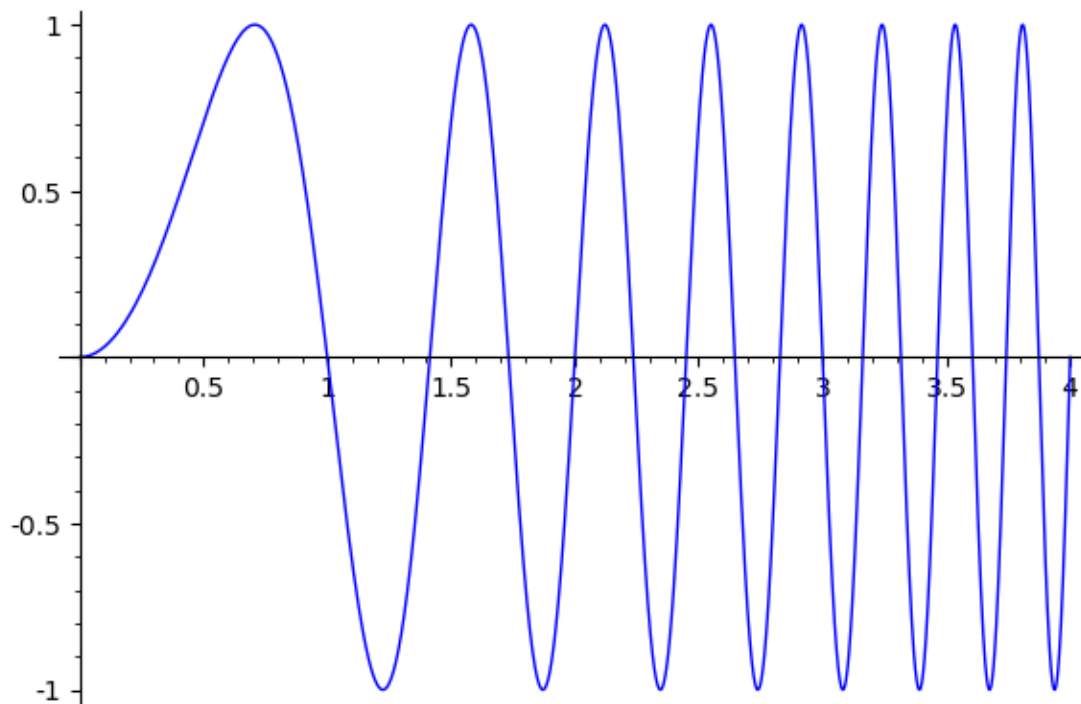
```
[3]: # 1c  
plot(sin(pi*x), 0, 4)
```

[3]:



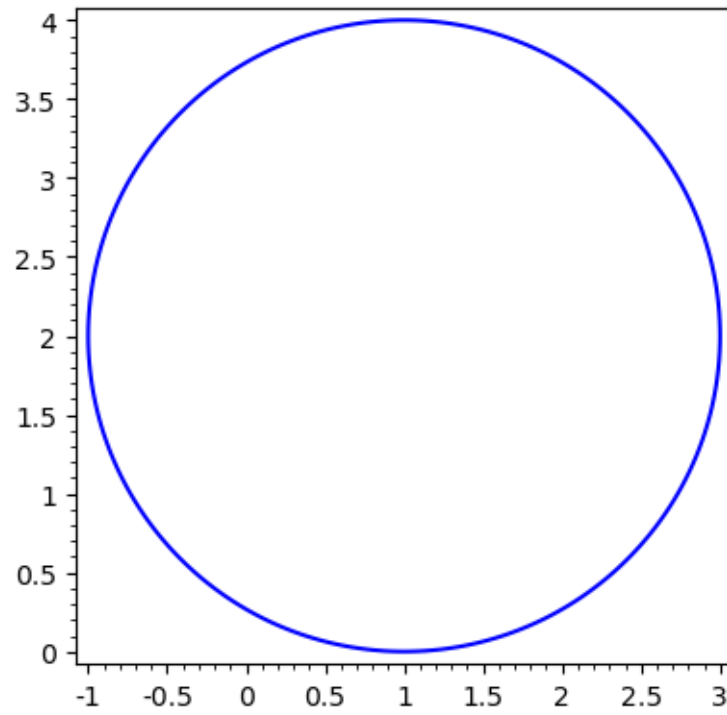
```
[4]: # 1d  
plot(sin(pi*x^2), 0, 4)
```

[4]:



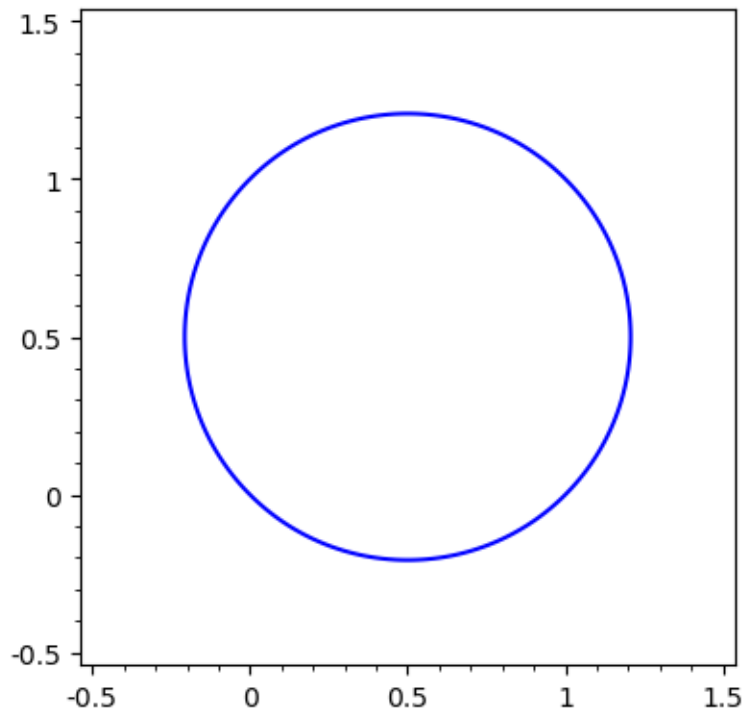
```
[5]: var("y") # we declare y to be a variable (x is assumed to be one)  
# 2a  
implicit_plot((x-1)^2 + (y-2)^2 == 4, (x,-1,3), (y,0,4))
```

[5]:



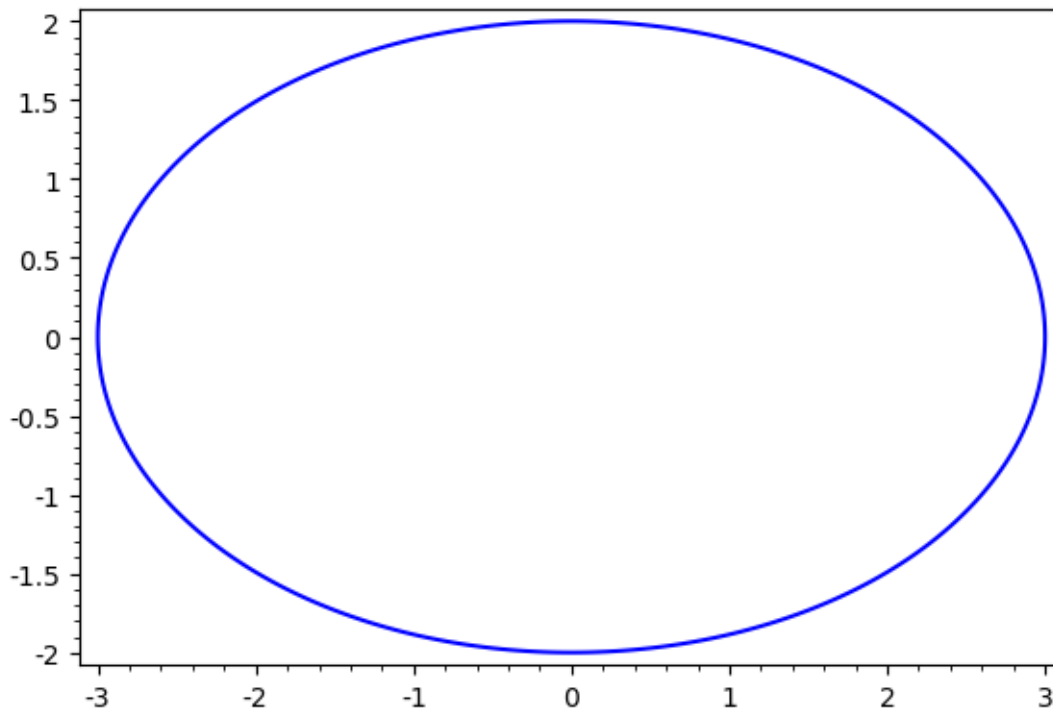
```
[6]: # 2b  
implicit_plot(x + y == x^2 + y^2, (x,-0.5,1.5), (y,-0.5,1.5))
```

[6]:



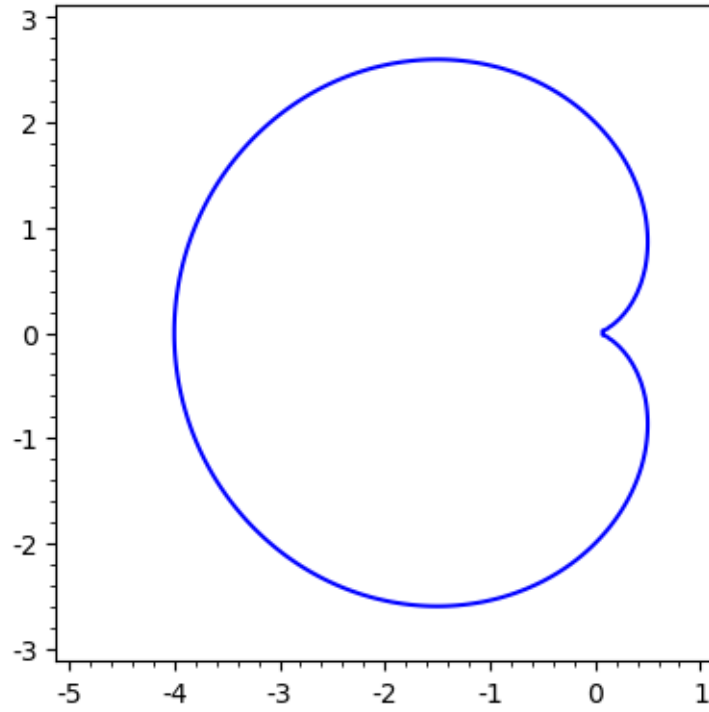
```
[7]: # 2c
      implicit_plot(4*x^2 + 9*y^2 == 36, (x,-3,3), (y,-2,2))
```

[7]:



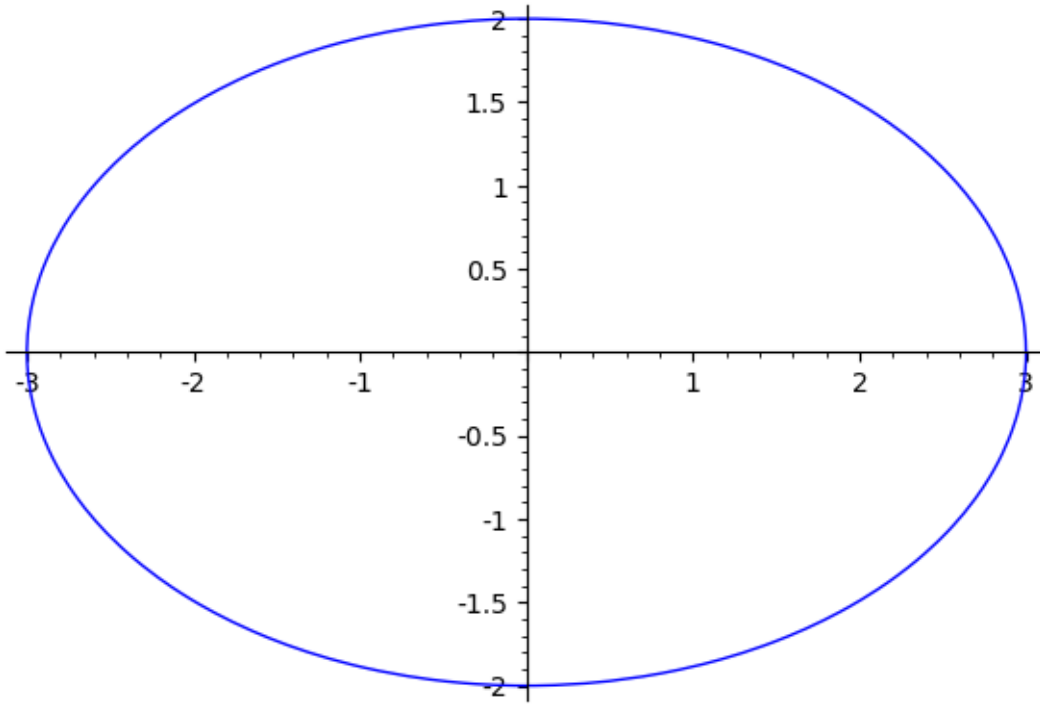
```
[8]: # 2d
      implicit_plot((x^2 + y^2)^2 + 4*x*(x^2 + y^2) - 4*y^2 == 0, (x,-5,1), (y,-3,3))
```

[8]:



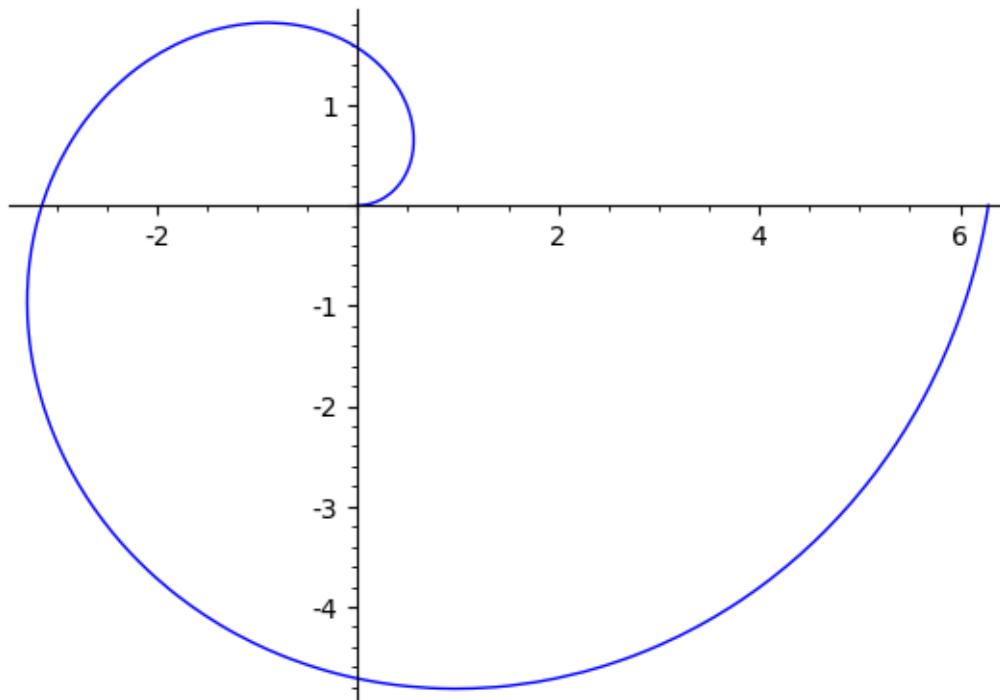
```
[9]: var("t") # we declare t to be a variable  
      # 3a  
      parametric_plot((3*cos(t), 2*sin(t)), (t,0,2*pi))
```

[9]:



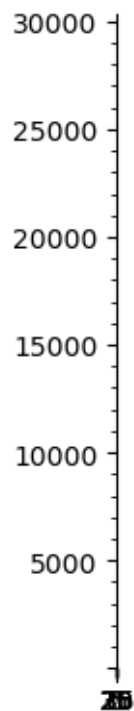
```
[10]: # 3b  
parametric_plot((t*cos(t), t*sin(t)), (t,0,2*pi))
```

[10]:



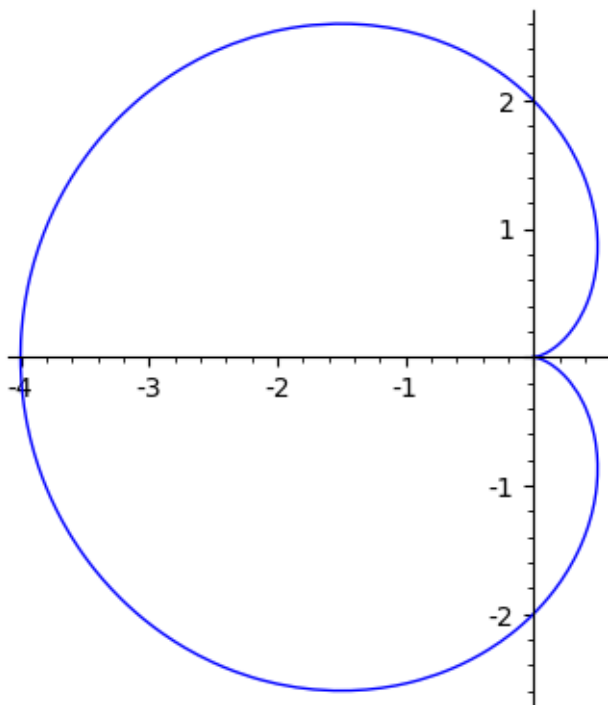
```
[11]: # 3c
parametric_plot((t^3, t^9 - t^3), (t,-1,1*pi))
# this was included as an example of SageMath's plotting failing to
# give a useful result on what ought to be innocuous input
```

[11]:



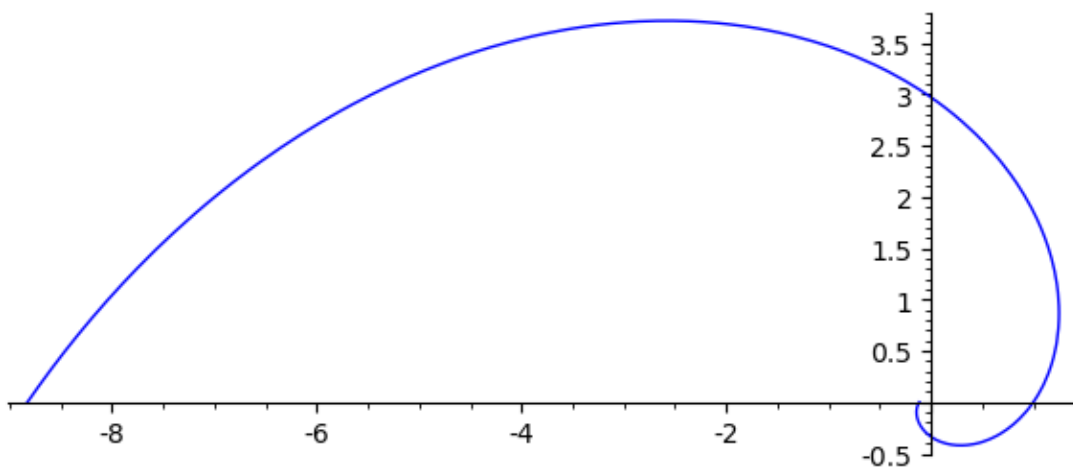
```
[12]: # 3d
parametric_plot((2*(1 - cos(t))*cos(t), 2*(1 - cos(t))*sin(t)), (t,0,2*pi))
```

[12]:



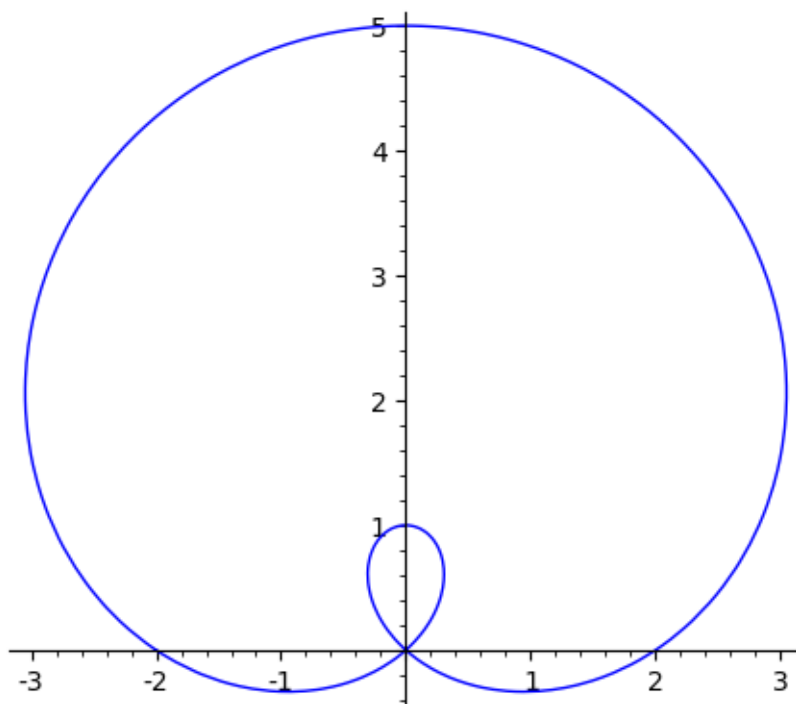
```
[14]: var("theta") # we declare theta to be a variable
      # 4a
      polar_plot(2^theta, (theta, -pi, pi))
```

[14]:



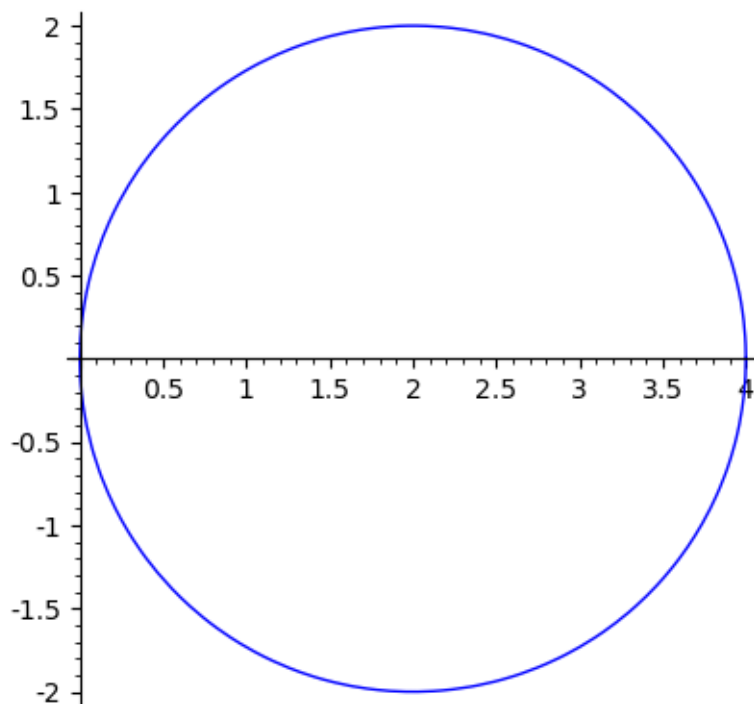
```
[16]: # 4b
      polar_plot(2+3*sin(theta), (theta, -pi, pi))
```

[16]:



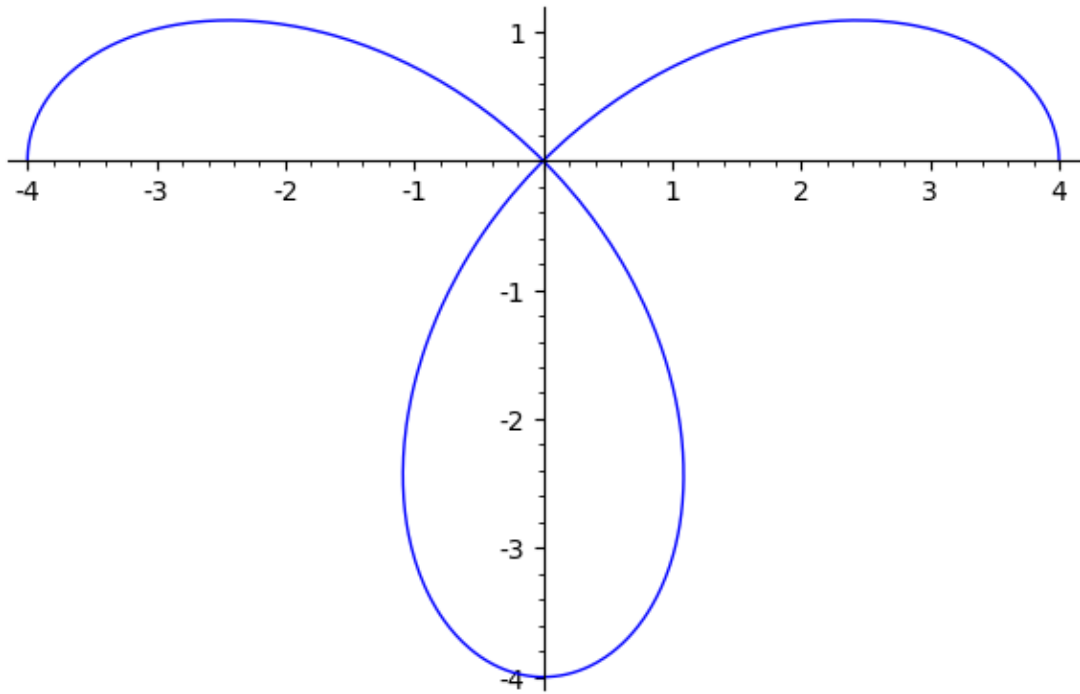
```
[17]: # 4c  
polar_plot(4*cos(theta), (theta,0,pi))
```

[17]:



```
[18]: # 4d  
polar_plot(4*cos(2*theta), (theta,0,pi))
```

[18]:



[]: