Mathematics 1120H – Calculus II: Integrals and Series

TRENT UNIVERSITY, Summer 2021 (S62)

Take-Home Final Examination

Released at noon on Wednesday, 28 July, 2021.

Due by noon on Saturday, 31 July, 2021.

INSTRUCTIONS

- You may consult your notes, handouts, and textbook from this course and any other math courses you have taken or are taking now. You may also use a calculator. However, you may not consult any other source, or give or receive any other aid, except for asking the instructor to clarify instructions or questions.
- Please submit an electronic copy of your solutions, preferably as a single pdf (a scan of handwritten solutions should be fine), via the Assignment module on Blackboard. If that doesn't work, please email your solutions to the intructor. *Show all your work!*
- Do all three (3) of Parts **X**, **Y**, and **Z**, and, if you wish, Part **B** as well.

Part X. Do both of **1** and **2**. *[40 = 2×20 each]*

1. Compute the integrals in any five (5) of **a** – **f**. *[20 = 5×4 each]*

a. $\displaystyle\int_0^{\pi/2} \sin(2x)\cos^2(x)\,dx$ **b.** $\displaystyle\int \frac{x+1}{x^3+x}\,dx$ **c.** $\displaystyle\int_1^e x\left(\ln(x)\right)^2\,dx$

d. $\displaystyle\int_0^{\pi/4} \tan^2(x)\sec^2(x)\,dx$ **e.** $\displaystyle\int \frac{\sqrt{1-x^2}}{\left(x^2-1\right)^2}\,dx$ **f.** $\displaystyle\int e^x\cosh(x)\,dx$

2. Determine whether the series converges in any five (5) of **a** – **f**. *[20 = 5×4 each]*

a. $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$ **b.** $\displaystyle\sum_{n=0}^{\infty} \frac{41^n}{n(n+1)}$ **c.** $\displaystyle\sum_{n=1}^{\infty} \frac{n!\cdot 2^n}{(2n)!}$

d. $\displaystyle\sum_{n=0}^{\infty} \frac{n^2-1}{\left(n^2+1\right)^2}$ **e.** $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!+2^n}$ **f.** $\displaystyle\sum_{n=100}^{\infty} \frac{\sin(n\pi)+\cos(n\pi)}{\ln\left(e^n\right)}$

Part Y. Do any three (3) of **3** – **6**. *[30 = 3×10 each]*

3. Sketch the solid obtained by revolving the region below $y = \sin(x)$ and above $y = -\sin(x)$, for $0 \le x \le \pi$, about the y-axis, and find its volume. *[10]*

4. Find the area of the surface obtained by revolving the curve $y = \dfrac{x^3}{3}$, for $0 \le x \le 1$, about the x-axis. *[10]*

5. Find the area of the region below $y = 0$ and above $y = \ln(x)$, where $0 < x \le 1$. *[10]*

6. Sketch the solid obtained by revolving the region below $y = \sin(x)$ and above $y = -1$, for $0 \le x \le 2\pi$, about the x-axis, and find its volume. *[10]*

*Parts **Z** and **B** are on the next page!*

*Parts **X** and **Y** are on the previous page!*

Part Z. Do any three (3) of **7** – **10**. *[30 = 3×10 each]*

7. Determine the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{n!}{2^n} x^n$. *[10]*

8. Consider the function $f(x) = \sin(x) + \sinh(x)$.

a. Use Taylor's formula to find the Taylor series centred at 0 of $f(x)$. *[4]*

b. Determine the radius and interval of convergence of this Taylor series. *[3]*

c. Find the Taylor series centred at 0 of $f(x)$ without using Taylor's formula. *[3]*

9. Find the Taylor series centred at π of $f(x) = \sin(x)$ and determine its radius and interval of convergence. *[10]*

10. Determine whether the series

$$\sum_{n=0}^{\infty} \left[\frac{1}{4n+1} + \frac{1}{4n+2} - \frac{1}{4n+3} - \frac{1}{4n+4} \right] = 1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{7} - \frac{1}{8} + \frac{1}{9} + \cdots$$

converges absolutely, converges conditionally, or diverges. If it is convergent, find its sum. *[10]*

[Total = 100]

Part B. ... is for bonus! If you want to, do one or both of the following problems.

α. Write a poem touching on calculus or mathematics in general. *[1]*

β. A certain mathematician once asserted that $1 + 2 + 4 + 8 + \cdots = -1$. What did this unfortunate person do to get this equation? *[1]*

I HOPE THAT YOU ENJOYED THE COURSE.
ENJOY THE REST OF THE SUMMER!