Mathematics 1120H – Calculus II: Integrals and Series
Trent University, Summer 2021 (S62)

Assignment #3
Kosh says its a cinch.
Due on Friday, 9 July.

Submission: Scanned or photographed solutions are fine, so long as they are legible. Please try to make sure that they are oriented correctly – if they are sideways or upside down, they're rather harder to mark online. Submission as a single pdf is strongly preferred, but other common formats are probably OK in a pinch. Please submit your solutions via Blackboard's Assignments module. If Blackboard does not acknowledge a successful upload, please try again. As a *last* resort, email your solutions to the instructor at: `sbilaniuk@trentu.ca`

The functions $\cosh(x) = \frac{1}{2}\left(e^x + e^{-x}\right)$ and $\sinh(x) = \frac{1}{2}\left(e^x - e^{-x}\right)$, mentioned in the lecture *Integration by Parts II*, are the basic *hyperbolic* functions, analogously to the basic trigonometric functions, $\cos(x)$ and $\sin(x)$. (The other hyperbolic functions are defined in terms of the basic ones in the same way that the other trigonometric functions are defined in terms of the basic one.) Their names are pronounced "kosh" and "sinch", respectively.

Since the basic hyperbolic functions are each others' derivatives (with no gratuitous negative signs) and satisfy a reasonably nice identity, namely $\cosh^2(x) - \sinh^2(x) = 1$ (sadly with a negative), they are sometimes used in place of the trigonometric functions when making substitutions. They have other uses in mathematics as well: they are needed to help do trigonometry in certain curved spaces, they arise in solving various differential equations (see question **3** below), and they turn out to be intimately related to the standard trigonometric functions. (For example, $\cos(x) = \cosh(ix)$ and $\cosh(x) = \cos(ix)$, where $i = \sqrt{-1}$.)

1. Verify that $\cosh^2(x) - \sinh^2(x) = 1$ for all x. *[1]*

2. Work out what the inverse function of $\sinh(x)$, let's call it $\operatorname{arcsinh}(x)$, is in terms of more common functions. *[3]*

3. Suppose $y = f(x)$ is a functions satisfying the the differential equation $\dfrac{d^2y}{dx^2} = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$, and also satisfies the initial conditions $f(1) = f(-1) = \cosh(1)$. Show that it must be the case that $f(x) = \cosh(x)$. *[6]*

Note the first: It is easy to check that $\cosh(x)$ satisfies the differential equation and the initial conditions. Why is it the only function that does?

Hint: Let $z = \dfrac{dy}{dx}$, so $\dfrac{dz}{dx} = \dfrac{d^2y}{dx^2}$. Rewrite the equation in terms of z, move everything involving x to one side of the new equation and everything involving z to the other), and integrate to solve for z. Then get y by ...

Note the second: This differential equation would arise if you suspended a certain length of chain from the points specified by the initial conditions and let it hang under the influence of gravity, "down" being the negative y direction, and asked what shape the chain would have if no other forces were in play.

[Total = 10]