Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Summer 2020 (S62)
Take-Home Final Examination
Released at noon on Wednesday, 29 July, 2020.
Due by noon on Saturday, 1 August, 2020.

INSTRUCTIONS

- You may consult your notes, handouts, and textbook from this course and any other math courses you have taken or are taking now. You may also use a calculator. However, you may not consult any other source, or give or receive any other aid, except for asking the instructor to clarify instructions or questions.
- Please submit an electronic copy of your solutions, preferably as a single pdf (a scan of handwritten solutions should be fine), via the Assignment module on Blackboard. If that doesn't work, please email your solutions to the intructor. *Show all your work!*
- Do all three (3) of Parts **I** – **III**, and, if you wish, Part **IV** as well.

Part I. Do both of **1** and **2**. *[40 = 2×20 each]*

1. Compute the integrals in any four (4) of **a** – **f**. *[20 = 4×5 each]*

a. $\displaystyle\int_0^{\pi/2} \sin(x)\sqrt{1+\cos^2(x)}\,dx$ **b.** $\displaystyle\int \frac{\ln(\ln(x))}{x}\,dx$ **c.** $\displaystyle\int \frac{x}{\sqrt{4-x^2}}\,dx$

d. $\displaystyle\int_{-1}^{1} \frac{1+\arctan^2(x)}{1+x^2}\,dx$ **e.** $\displaystyle\int_0^1 x\arctan(x)\,dx$ **f.** $\displaystyle\int \frac{1}{\sqrt{4+x^2}}\,dx$

2. Determine whether the series converges in any four (4) of **a** – **f**. *[20 = 4×5 each]*

a. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n-3^n}{4^n+(-1)^n}$ **b.** $\displaystyle\sum_{n=0}^{\infty} (-3)^{-n}e^n$ **c.** $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

d. $\displaystyle\sum_{n=0}^{\infty} \frac{\sin(n)+\cos(n)}{n^3+n^2+n+1}$ **e.** $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$ **f.** $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

Part II. Do any three (3) of **3** – **6**. *[30 = 3×10 each]*

3. Find the volume of the solid obtained by revolving the region below $y = 4 - x^2$ and above $y = 0$, for $-2 \le x \le 2$, about the x-axis. *[10]*

4. a. Find the arc-length of the curve $y = \ln(\cos(x))$, where $0 \le x \le \frac{\pi}{4}$. *[6]*

b. Find the average value of $\tan(x)$ on the interval $\left[0, \frac{\pi}{4}\right]$. *[4]*

5. Find the area of the surface obtained by revolving the curve $y = \sin(x)$, for $0 \le x \le \pi$, about the x-axis. *[10]*

6. Work out $\displaystyle\int \frac{x^3-x^2+x+59}{x^3-x^2+x-1}\,dx$. *[10]*

More exam on page 2!

Part III. Do any three (3) of **7** – **10**. *[30 = 3×10 each]*

7. Determine the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{2n}$. What function has this power series as its Taylor series at 0? *[10]*

8. Consider the rational function $q(x) = \dfrac{x^7 - 1}{x - 1}$. Find the Taylor series at 0 of $q(x)$ and determine its radius and interval of convergence. *[10]*

9. Find the Taylor series at 0 of $f(x) = \dfrac{1}{3 + x}$ and determine its radius and interval of convergence. *[10]*

10. In each case, give an example (or explain why there isn't one) of a series $\displaystyle\sum_{n=2}^{\infty} a_n$

- **a.** … that diverges, but $\displaystyle\sum_{n=2}^{\infty} (-1)^n a_n$ converges. *[1]*
- **b.** … that converges, but $\displaystyle\sum_{n=2}^{\infty} (-1)^n a_n$ diverges. *[1]*
- **c.** … that diverges, but $\displaystyle\sum_{n=2}^{\infty} a_n^2$ converges. *[2]*
- **d.** … that converges, but $\displaystyle\sum_{n=2}^{\infty} a_n^2$ diverges. *[2]*
- **e.** … that converges conditionally, but $\displaystyle\sum_{n=2}^{\infty} (-1)^n a_n$ converges absolutely. *[2]*
- **f.** … that converges absolutely, but $\displaystyle\sum_{n=2}^{\infty} (-1)^n a_n$ converges conditionally. *[2]*

[Total = 100]

Part IV. Bonus! If you want to, do one or both of the following problems.

41. Write a poem touching on calculus or mathematics in general. *[1]*

42. When does $6 \times 9 = 42$ actually work? (With apologies to Douglas Adams. :-) *[1]*

THANK YOU ALL FOR BEARING WITH THE COURSE UNDER DIFFICULT CIRCUMSTANCES.
ENJOY THE REST OF THE SUMMER!