Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Winter 2020

Assignment #4
A Little Series Algebra
Due on Friday, 17 July.

Please submit your solutions using Blackboard's assignment module. If that fails, please email your solutions to the instructor (`sbilaniuk@trentu.ca`). Scans or photos of handwritten solutions are perfectly acceptable, so long as they are legible and in some common format. (Combined into a single pdf, for preference.)

The series $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$ is a geometric series with first term $a = 1$ and common ratio $r = x$, and so adds up to $\frac{a}{1-r} = \frac{1}{1-x}$ when $|r| = |x| < 1$. For questions **1** and **2** you may assume that $|x| < 1$, so that the series adds up nicely.

1. Find a series $\sum_{n=0}^{\infty} a_n x^n$ such that $\sum_{n=0}^{\infty} a_n x^n = \left(\sum_{n=0}^{\infty} x^n \right)^2$. *[4]*

2. Find a series $\sum_{n=0}^{\infty} b_n x^n$ such that $\left(\sum_{n=0}^{\infty} x^n \right) \left(\sum_{n=0}^{\infty} b_n x^n \right) = 1$. *[1]*

Recall from Assignment #3 that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$. This series actually converges for all x, as we shall see later.

3. Find a series $\sum_{n=0}^{\infty} c_n x^n$ such that $\sum_{n=0}^{\infty} c_n x^n = \left(\sum_{n=0}^{\infty} \frac{x^n}{n!} \right)^2$. *[3]*

4. Find a series $\sum_{n=0}^{\infty} d_n x^n$ such that $\left(\sum_{n=0}^{\infty} \frac{x^n}{n!} \right) \left(\sum_{n=0}^{\infty} d_n x^n \right) = 1$. *[2]*