Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals
Trent University, Summer 2018
Actual Final Examination

Time-space: *09:00-12:00 in FPHL 117.* *Brought to you by* Стефан Біланюк.
Instructions: Do parts **A** and **B**, and, if you wish, part **C**. Show all your work and justify all your answers. *If in doubt about something,* **ask!**
Aids: Any calculator; (all sides of) one aid sheet; one (1) brain (no neuron limit).

Part A. Do all four (4) of **1–4**.

1. Compute $\frac{dy}{dx}$ as best you can in any *four* (4) of **a–f**. *[20 = 4 × 5 each]*

a. $y = 3^x$ **b.** $x^3 - y^2 = 0$ **c.** $y = x \cdot \left[\int_1^x t^2\, dt\right]$

d. $y = \dfrac{x}{x^2+2}$ **e.** $y = e^x \cos(x)$ **f.** $y = \tan^2(x)$

2. Evaluate any *four* (4) of the integrals **a–f**. *[20 = 4 × 5 each]*

a. $\displaystyle\int x \arctan(x)\, dx$ **b.** $\displaystyle\int_0^{\pi/4} \cos(2t)\, dt$ **c.** $\displaystyle\int_e^{e^e} \frac{1}{w\ln(w)}\, dw$

d. $\displaystyle\int \frac{1}{(2y+1)^2}\, dy$ **e.** $\displaystyle\int z\tan(z)\, dz$ **f.** $\displaystyle\int_0^1 4u\, e^{u^2}\, du$

3. Do any *four* (4) of **a–f**. *[20 = 4 × 5 each]*

a. Find the equation of the tangent line to $y = \sin(x)$ at $x = \frac{\pi}{2}$.

b. Compute $\displaystyle\lim_{x\to\infty} \frac{\ln\left(x^2\right)}{x}$.

c. Use the limit definition of the derivative to verify that $\frac{d}{dx}e^x = e^x$ for all x.

[You may assume that $\displaystyle\lim_{h\to 0} \frac{e^h - 1}{h} = 1$.]

d. Find the minimum value of $f(x) = xe^x$, if it has one.

e. Use the ε–δ definition of limits to verify that $\displaystyle\lim_{x\to 3}(4-x) = 1$.

f. Sketch the region between $y = x^3$ and $y = x$ for $-1 \le x \le 0$ and find its area.

4. Find the domain and all intercepts, vertical and horizontal asymptotes, and maximum, minimum, and inflection points of $f(x) = \dfrac{x^2+1}{x}$, and sketch its graph. *[14]*

Part B. Do any *two* (2) of **5–7**. *[28 = 2 × 13 each]*

5. A pebble is dropped into a still pool of water, creating a circular ripple that moves out from the point of impact at a constant rate of 2 m/s. How are the total length of the ripple and the area enclosed by the ripple changing after 3 s?

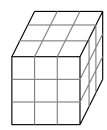
6. Consider the region in the first quadrant (*i.e.* where both $x \geq 0$ and $y \geq 0$) below $y = 4 - x$, and above both $y = 4 - 3x$ and $y = x^2 - 2x + 2$. Find the coordinates of the three corners of this region, sketch this region, and compute the area of this region.

7. What is the maximum area of a triangle whose vertices are the points $(0, 0)$, $(x, 0)$, and $\left(x, \dfrac{1}{1+x^2}\right)$ for some $x \geq 0$?

[Total = 100]

Part C. Bonus problems! If you feel like it and have the time, do one or both of these.

□. A dangerously sharp tool is used to cut a cube with a side length of $3\,cm$ into 27 smaller cubes with a side length of $1\,cm$. This can be done easily with six cuts. Can it be done with fewer? (Rearranging the pieces between cuts is allowed.) If so, explain how; if not, explain why not. *[1]*



△. Write a haiku touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

ENJOY THE REST OF THE SUMMER!