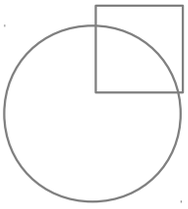
**Mathematics 1100Y – Calculus I: Calculus of one variable**
TRENT UNIVERSITY, Summer 2012

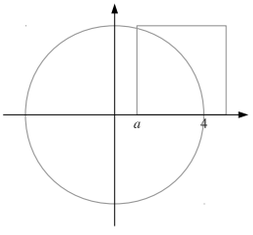
**Solution to Assignment #8**
**Circling half a square**



**1.** A square with sides of length 4 is moved into a circle of radius 4 so that one side of the square slides along a radius of the circle until exactly half the area of the square is inside the circle. How far is the corner of the square inside the circle from the centre of the circle at this point? (Give a number!) *[10]*

HINT: Set up an integral equation for the overlapping area that has the desired distance as an unknown constant, then use `Maple` to solve the equation for that constant. You may want to look up `Maple`' `int`s operator and the `fsolve` command, though there are other ways to get `Maple` to do the job.

SOLUTION. For convenience, we'll arrange our coordinates so that the circle is centred at the origin, the square is in the first quadrant, and the side of the square lying along a radius is along the $x$-axis. Let $a$ be the distance from the corner of the square inside the circle to the origin.




The part of the circle in the first quadrant has equation $y = \sqrt{16 - x^2}$, from which it follows that the area of the part of the square inside the circle is given by the following integral:

$$\int_a^4 \sqrt{16 - x^2}\, dx$$

We set this equal to half area of the square, *i.e.* to $\frac{1}{2} \cdot 4^2 = \frac{16}{2} = 8$, and let `Maple` do the rest:

```
> fsolve(int(sqrt(1-x^2),x=a..4)=8,a)
```

1.157976732

This is, of course, merely an approximation. An exact expression that is useful is pretty hard to come by here, though.

The suspiciously-minded may notice that, to the same number of decimal places, $\frac{\pi}{2} = 1.570963267$. Not quite the same, but with some similarities. Coincidence? Maybe, maybe not ... ■