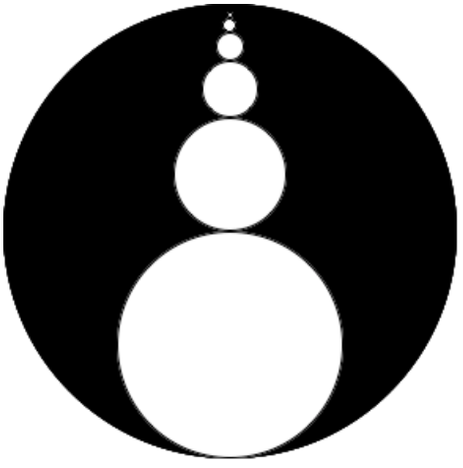
Mathematics 1100Y – Calculus I: Calculus of one variable
TRENT UNIVERSITY, Summer 2012

Assignment #1
Designs for a (non-Olympic) diskus?!
Due on Wednesday, 23 May, 2012.

Consider the shape obtained as follows:

0. Start with a disk of radius 1.
1. Remove a disk of radius $\frac{1}{2}$ that just touches the centre and the edge of the larger disk.
2. Remove a disk of radius $\frac{1}{4}$ whose centre is on the straight line defined by the centres of the previous disks and which just touches the disk removed at step 1.
3. Remove a disk of radius $\frac{1}{8}$ whose centre is on the straight line defined by the centres of the previous disks and which just touches the disk removed at step 2.
4. Remove a disk of radius $\frac{1}{16}$ whose centre is on the straight line defined by the centres of the previous disks and which just touches the disk removed at step 3.
$\vdots$
n. Remove a disk of radius $\frac{1}{2^n}$ whose centre is on the straight line defined by the centres of the previous disks and which just touches the disk removed at step $n-1$.
$\vdots$

The object obtained after the first few steps of this process is illustrated below:



1. Find a formula (in terms of n) for the area of the shape obtained at step n. *[4]*
Note: Just in case, the area of a circle of radius r is πr^2 ...

2. What is the area of the shape obtained after infinitely many steps? *[1]*

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Now consider the shape obtained as follows:

0. Start with a disk of radius 1.

1. Remove a disk of radius $\frac{1}{2}$ that just touches the centre and the edge of the larger disk.

2. Add back a disk of radius $\frac{1}{4}$ whose centre is on the straight line defined by the centres of the previous disks and which just touches both previous disks.

3. Remove a disk of radius $\frac{1}{8}$ whose centre is on the straight line defined by the centres of the previous disks and which just touches all the previous disks.

4. Add back a disk of radius $\frac{1}{16}$ whose centre is on the straight line defined by the centres of the previous disks and which just touches all the previous disks.

$\vdots$

2k+1. Remove a disk of radius $\frac{1}{2^{2k+1}}$ whose centre is on the straight line defined by the centres of the previous disks and which just touches all the previous disks.

$\vdots$

2k+2. Add back a disk of radius $\frac{1}{2^{2k+2}}$ whose centre is on the straight line defined by the centres of the previous disks and which just touches all the previous disks.

$\vdots$

The object obtained after the first few steps of this process is illustrated below:

3. Find a formula (or formulas) for the area of the shape obtained at step n (or steps $2k+1$ and $2k+2$). *[4]*

4. What is the area of the shape obtained after infinitely many steps? *[1]*