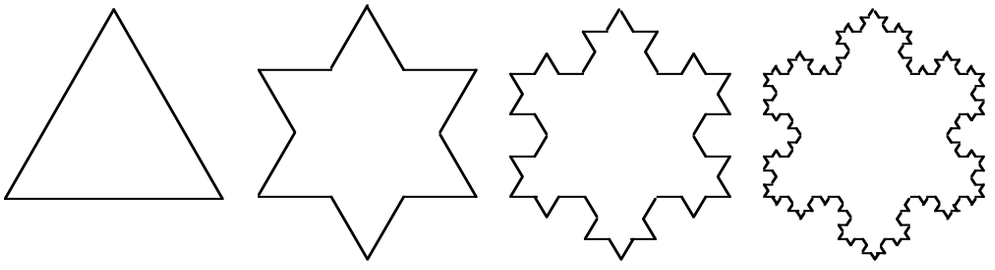
Mathematics 1100Y – Calculus I: Calculus of one variable
TRENT UNIVERSITY, Summer 2010

Assignment #1
The Snowflake Curve*
Due on Wednesday, 19 May, 2010.

Suppose one has an equilateral triangle with sides of length 1. If one modifies each of the line segments composing the triangle by cutting out the middle third of the segment, and then inserting an outward-pointing "tooth," both of whose sides are as long as the removed third, one gets a six-pointed star. Suppose one repeats this process for each of the line segments making up the star, then to each of the line segments making up the resulting figure, and so on, as in the diagram:



Note that the lengths of the line segments at each stage are a third of the length of the segments at the preceding stage. For the sake of being definite, let's say we have the triangle at step 0 of the process, the six-pointed star at step 1 of the process, the next shape at step 2 of the process, and so on. The curve which is the limit of this process, if one takes infinitely many steps, is often called the *snowflake curve**. We will try to discover the length of this curve and the area of the region that it encloses below.

1. What are the lengths of the curves we have at steps 0, 1, 2, and 3 of the process? *[1]*

2. What is the length of the curve at step n of the process? *[1]*

3. Suppose $L > 0$ is some positive number. Find an number N such that if $n \geq N$, then the length of the curve at step n is $\geq L$. *[2]*

4. What does **3** suggest about the length of the snowflake curve? *[1]*

5. What are the areas of the shapes we have at steps 0, 1, 2, and 3 of the process? *[1]*

6. What is the area of the shape at step n of the process? *[1]*

7. What is the difference between the area of the shape we have at step n and $\frac{3\sqrt{3}}{4}$? *[2]*

8. What does your answer to **7** suggest about the area of the region enclosed by the snowflake curve? *[1]*

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* Also known as the *Koch curve.*