Mathematics 1110H (Section B) – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2023

Solutions to Quiz #4
Derivatives

REMINDER. While you are allowed to work together and look things up when doing the quizzes and assignments, your submission should be written up entirely by yourself, giving credit to any collaborators or sources that you ended up actually using. Please show all your steps and simplify your answers as far as practical.

1. Using the practical rules for computing derivatives, find $\frac{d}{dx}\sqrt{x}$ for $x > 0$. *[1]*

SOLUTION. We will use the Power Rule for differentiation:

$$\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{1/2} = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2}\cdot\frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Note that since $x > 0$, we are not at risk of dividing by 0 in the final answer. □

2. Using the practical rules for computing derivatives, find $\frac{d}{dx}e^{-x^2/2}$. *[1.5]*

SOLUTION. We will use the Chain Rule and the Power Rule for differentiation:

$$\begin{aligned}\frac{d}{dx}e^{-x^2/2} &= e^{-x^2/2}\frac{d}{dx}\left(-\frac{x^2}{2}\right) = e^{-x^2/2}\left(-\frac{1}{2}\right)\frac{d}{dx}x^2 = -\frac{1}{2}e^{-x^2/2}\cdot 2x \\ &= -xe^{-x^2/2} \quad \square\end{aligned}$$

3. Using the limit definition of the derivative and the practical rules for computing limits, find $\frac{d}{dx}\sqrt{x}$ for $x > 0$. *[2.5]*

SOLUTION. Our main tool will be a little bit of algebraic trickery, running a difference of squares in reverse.

$$\begin{aligned}\frac{d}{dx}\sqrt{x} &= \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} \\ &= \lim_{h\to 0}\frac{\left(\sqrt{x+h}\right)^2-\left(\sqrt{x}\right)^2}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \lim_{h\to 0}\frac{(x+h)-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)} \\ &= \lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}} \\ &= \frac{1}{\sqrt{x+0}+\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}\end{aligned}$$

As in the solution to **1** above, we are not at risk of dividing by 0 in the final answer because $x > 0$. □