

# Mathematics 1110H (Section B) – Calculus I: Limits, Derivatives, and Integrals

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## Solutions to Quiz #3

### Limits

REMINDER. While you are allowed to work together and look things up when doing the quizzes and assignments, your submission should be written up entirely by yourself, giving credit to any collaborators or sources that you ended up actually using.

1. Using the practical rules for computing limits, find  $\lim_{x \rightarrow -3} \frac{4x^2 - 36}{x + 3}$ . [2.5]

SOLUTION. Here we go:

$$\begin{aligned} \lim_{x \rightarrow -3} \frac{4x^2 - 36}{x + 3} &= \lim_{x \rightarrow -3} \frac{4(x^2 - 9)}{x + 3} = 4 \lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3} = 4 \lim_{x \rightarrow -3} \frac{(x - 3)(x + 3)}{x + 3} \\ &= 4 \lim_{x \rightarrow -3} (x - 3) = 4(-3 - 3) = 4(-6) = -24 \quad \square \end{aligned}$$

2. Use the  $\varepsilon$ - $\delta$  definition of limits to verify that your answer to question 1 is correct. [2.5]

SOLUTION. We need to use the  $\varepsilon$ - $\delta$  definition of limits to verify that  $\lim_{x \rightarrow -3} \frac{4x^2 - 36}{x + 3} = -24$ ;

that is, we need to check that for every  $\varepsilon > 0$  there is a  $\delta > 0$  such that if  $|x - (-3)| < \delta$ ,

then  $\left| \frac{4x^2 - 36}{x + 3} - (-24) \right| < \varepsilon$ . As usual, we will try to reverse-engineer the necessary  $\delta$  from  $\left| \frac{4x^2 - 36}{x + 3} - (-24) \right| < \varepsilon$ .

$$\begin{aligned} \left| \frac{4x^2 - 36}{x + 3} - (-24) \right| < \varepsilon &\iff \left| \frac{4(x^2 - 9)}{x + 3} + 24 \right| < \varepsilon \iff \left| \frac{4(x - 3)(x + 3)}{x + 3} + 24 \right| < \varepsilon \\ &\iff |4(x - 3) + 24| < \varepsilon \iff |4x - 12 + 24| < \varepsilon \\ &\iff |4x + 12| < \varepsilon \iff |4(x + 3)| < \varepsilon \iff 4|x + 3| < \varepsilon \\ &\iff |x + 3| < \frac{\varepsilon}{4} \iff |x - (-3)| < \frac{\varepsilon}{4} \end{aligned}$$

If we now set  $\delta = \frac{\varepsilon}{4}$ , then, since every step above is reversible, if  $|x - (-3)| < \delta = \frac{\varepsilon}{4}$ , then

$$\left| \frac{4x^2 - 36}{x + 3} - (-24) \right| < \varepsilon.$$

By the  $\varepsilon$ - $\delta$  definition of limits, it follows that  $\lim_{x \rightarrow -3} \frac{4x^2 - 36}{x + 3} = -24$ .  $\square$