Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals (Section C)
TRENT UNIVERSITY, Fall 2021

Assignment #1
Games With Limits
Due on Friday, ~~17~~ 24 September.

Submission: Scanned or photographed handwritten solutions are fine, so long as they are legible. Submission as a single pdf is strongly preferred, but other common formats are probably OK. (If not, we'll get back to you! :-) Please submit via Blackboard's Assignments module. If that fails, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`

The usual $\varepsilon - \delta$ definition of limits,

> DEFINITION. $\lim_{x \to a} f(x) = L$ exactly when for every $\varepsilon > 0$ there is a $\delta > 0$ such that for any x with $|x - a| < \delta$ we are guaranteed to have $|f(x) - L| < \varepsilon$ as well.

is pretty hard to wrap your head around the first time or three for most people. Here is less common version of the definition, equivalent to the standard one, which recasts the confusing logical structure of the standard definition in terms of a game:

> ALTERNATE DEFINITION. The *limit game* for $f(x)$ at $x = a$ with target L is a three-move game played between two players A and B as follows:
>
> 1. A moves first, picking a small number $\varepsilon > 0$.
> 2. B moves second, picking another small number $\delta > 0$.
> 3. A moves third, picking an x that is within δ of a, *i.e.* $a - \delta < x < a + \delta$.
>
> To determine the winner, we evaluate $f(x)$. If it is within ε of the target L, *i.e.* $L - \varepsilon < f(x) < L + \varepsilon$, then player B wins; if not, then player A wins.
>
> With this idea in hand, $\lim_{x \to a} f(x) = L$ means that player B has a winning strategy in the limit game for $f(x)$ at $x = a$ with target L; that is, if B plays it right, B will win no matter what A tries to do. (Within the rules ... :-) Conversely, $\lim_{x \to a} f(x) \neq L$ means that player A is the one with a winning strategy in the limit game for $f(x)$ at $x = a$ with target L.

The game definition of limits isn't really better or worse that the usual $\varepsilon - \delta$ definition, but each is easier for some people to understand, and the exercise in trying it both ways usually helps in understanding what is really going on with limits.

1. Use one of these two definitions of limit to verify that $\lim_{x \to -2} (3x + 4) = 2$. *[2.5]*

2. Use the definition of limit that you didn't use in answering question **1** to verify that $\lim_{x \to 1} (x + 6) \neq 9$. *[2.5]*

3. Use either definition of limits above to verify that $\lim_{x \to 0} \sin(2x) = 0$. *[2.5]*

Hint: You may use the fact that $|\sin(t)| \leq |t|$ for all real numbers t.

4. Use either definition of limits above to verify that $\lim_{x \to 3} x^2 = 9$. *[2.5]*

Hint: The choice of δ in **4** will probably require some indirect reasoning. Pick some arbitrary smallish positive number for δ as a first cut. If it doesn't do the job, but x is at least that close, you'll have more information to help pin down the δ you really need.