Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals
Trent University, Fall 2020

Solutions to Quiz #10

Tuesday, 1 December .

Scans or photos of handwritten work are entirely acceptable so long as they are legible and in some common format; solutions submitted as a single pdf are preferred, if you can manage it. If you can't submit your solutions on time via Blackboard's Assignments module for some reason, please email them to the instructor at: `sbilaniuk@trentu.ca`

Compute each of the following integrals.

1. $\displaystyle\int \left(e^x - e^{-x}\right)\,dx$ *[1.5]*

Solution. Here goes:

$$\int \left(e^x - e^{-x}\right)\,dx = \int e^x\,dx - \int e^{-x}\,dx \qquad \begin{array}{l}\text{Substitute } u=-x \text{ in the second integral,}\\ \text{so } du=(-1)dx \text{ and } dx=(-1)du.\end{array}$$
$$= e^x - \int e^u\,(-1)du = e^x - (-1)e^u + C = e^x + e^{-x} + C \qquad \square$$

2. $\displaystyle\int x^3\sqrt{1+x^2}\,dx$ *[1.5]*

Solution. We will use the substitution $w = 1+x^2$, so $dw = 2x\,dx$ and $x\,dx = \frac{1}{2}\,dw$, and also $x^2 = w-1$.

$$\begin{aligned}\int x^3\sqrt{1+x^2}\,dx &= \int x\cdot x^2\sqrt{1+x^2}\,dx = \int (w-1)\sqrt{w}\,\frac{1}{2}\,dw\\ &= \frac{1}{2}\int (w-1)w^{1/2}\,dw = \frac{1}{2}\int \left(w^{3/2}-w^{1/2}\right)\,dw\\ &= \frac{1}{2}\left(\frac{w^{5/2}}{5/2}-\frac{w^{3/2}}{3/2}\right)+C = \frac{1}{5}w^{5/2}-\frac{1}{3}w^{3/2}+C\\ &= \frac{1}{5}\left(1+x^2\right)^{5/2}-\frac{1}{3}\left(1+x^2\right)^{3/2}+C \qquad \square\end{aligned}$$

3. $\displaystyle\int_0^{\pi/2} \frac{\cos(x)}{1+\sin^2(x)}\,dx$ *[2]*

Solution. We will use the substitution $z = \sin(x)$, so $dz = \cos(x)\,dx$, and change the limits as we go along: $\begin{array}{ccc} x & 0 & \pi/2 \\ z & 0 & 1 \end{array}$ [Recall that $\sin(0) = 0$ and $\sin(\pi/2) = 1$.]

$$\begin{aligned}\int_0^{\pi/2} \frac{\cos(x)}{1+\sin^2(x)}\,dx &= \int_0^1 \frac{1}{1+z^2}\,dz = \arctan(z)\Big|_0^1\\ &= \arctan(1)-\arctan(0) = \frac{\pi}{4}-0 = \frac{\pi}{4} \qquad \blacksquare\end{aligned}$$