Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2020
Take-Home Final Examination for Section A

Available on Blackboard from 12:00 a.m. on Tuesday, 15 December.
Due on Blackboard by 11:59 p.m. on Thursday, 17 December.

Submission: Scans or photos of handwritten work are entirely acceptable so long as they are legible and in some common format; solutions submitted as a single pdf are strongly preferred. If submission via Blackboard's Assignments module fails repeatedly, then (only as a *last* resort) email them to the instructor at: `sbilaniuk@trentu.ca`

Allowed aids: For this exam, you are permitted to use your textbook and all other course material, from this and any other mathematics course(s) you have taken or are taking now, but *you may not use any other sources or aids, nor give or receive any help*, except to ask the instructor to clarify questions and to use a calculator (any that you like).

Instructions: Do parts **U** and **V**, and, if you wish, part **W**. Please show all your work and justify all your answers. *If in doubt about something,* **ask!**

Part U. Do *all four* (4) of **1**–**4**. *[Subtotal = 74]*

1. Compute $\frac{dy}{dx}$ as best you can in any *five* (5) of **a**–**f**. *[20 = 5 × 4 each]*

a $y = \dfrac{e^x}{x+1}$ **b.** $y = \displaystyle\int_0^{\ln(x)} e^{2t}\,dt$ **c.** $y = e^{-x^2}$

d. $y = 3^{\sin(x)}$ **e.** $\ln\left(x^2+y^2\right) = 0$ **f.** $y = \sqrt{x\tan(x)}$

2. Evaluate any *five* (5) of the integrals **a**–**f**. *[20 = 5 × 4 each]*

a. $\displaystyle\int xe^{x-1}\,dx$ **b.** $\displaystyle\int_0^{\pi/8} \tan(2y)\,dy$ **c.** $\displaystyle\int e^z \ln\left((e^z+1)^2\right)\,dz$

d. $\displaystyle\int_0^3 (w-1)(w+3)\,dw$ **e.** $\displaystyle\int \frac{v+\arctan(v)}{1+v^2}\,dv$ **f.** $\displaystyle\int_0^2 \frac{4u}{\sqrt{4+u^2}}\,du$

3. Do any *five* (5) of **a**–**g**. *[20 = 5 × 4 each]*

a. Consider the parametric curve given by $x = \cos(t)$ and $y = \sin(t)$ for $0 \le t \le \pi$. What is the slope of the tangent line to the curve when $t = \frac{\pi}{4}$?

b. Compute the area of the finite region between $y = \sqrt{x}$ and $y = \frac{x}{2}$.

c. Compute $\displaystyle\lim_{x\to\infty} e^{-x}\ln(x)$.

d. Use the limit definition of the derivative to show that $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$ for all $x > 0$.

Sadly, there is more on page 2 ...

. . . and here it is:

e. Find the absolute minimum value of $f(x) = \arctan\left(x^2\right)$ on $(-\infty, \infty)$.

f. Use the ε–δ definition of limits to verify that $\lim_{x \to -2} x^2 = 4$.

g. Find the volume of the solid obtained by revolving the region below $y = 3$ and above $y = 4 - x$, for $1 \le x \le 3$ about the x-axis.

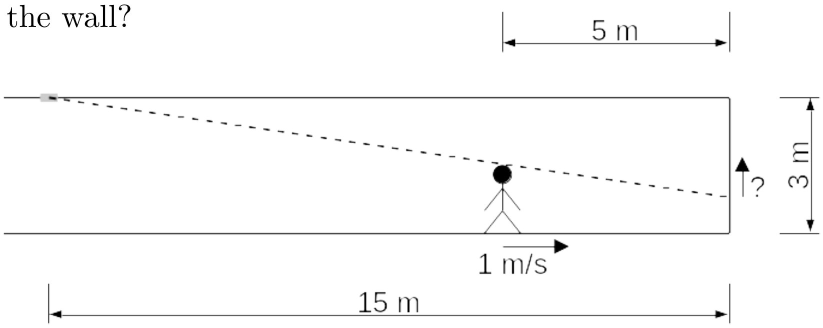
4. Find the domain as well as any (and all) intercepts, vertical and horizontal asymptotes, intervals of increase, decrease and concavity, and maximum, minimum, and inflection points of $h(x) = e^{-x^2/2}$, and sketch its graph based on this information. *[14]*

Part V. Do any *two* (2) of **5**–**7**. *[Subtotal = 26 = 2 × 13 each]*

5. Sketch the solid obtained by revolving the region between $y = x^2 - 2$ and $y = x$, where $0 \le x \le 2$, about the y-axis, and find its volume.

6. A triangle has one vertex at the point $(0, -4)$ and the other two at the points $(-x, y)$ and (x, y), where $x^2 + y^2 = 16$. Find the maximum area of such a triangle.

7. A straight and narrow corridor has a ceiling 3 m above the floor. The only illumination is from a ceiling panel 15 m from where the corridor ends in a vertical wall. Stick Person, who is 1.5 m tall, walks past the lamp towards the blank wall at 1 m/s. How quickly is the top of the shadow Stick casts on the wall rising at the instant that Stick is 5 m from the wall?



[Total = 100]

Part W. *Bonus problems!* If you feel like it, do one or both of these.

2^3. Find a formula in terms of n for the sum $\sum_{i=1}^{n} i(i+4) = 1 \cdot 5 + 2 \cdot 6 + \cdots + n(n+4)$. *[1]*

3^2. Write a haiku touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

WE HOPE THAT YOU ENJOYED THE COURSE. ENJOY THE BREAK!