Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals
Trent University, Fall 2020

Assignment #4
Plotting with Cartesian and Polar Coordinates
Due on Friday, 13 November.

Submission: Scanned or photographed solutions are fine, so long as they are legible. Submission as a single pdf is strongly preferred, but other common formats are probably OK. Please submit your solutions via Blackboard's Assignments module. If that fails, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`

Before attempting the questions below, please at least skim through Chapter 1 of the text for the basics of graphing functions in Cartesian coordinates, and Chapter 10 for the basics of parametric curves and polar coordinates. Some basics of graphing in Cartesian coordinates, using trigonometric functions, and so on, can also be found in the Academic Skills pamphlet *Formula for Success*. The basic definitions of how parametric curves and polar coordinates work are given in this assignment for your convenience, but you might want some additional explanations and examples.

Besides getting you a little bit acquainted with parametric curves and polar coordinates, the purpose of this assignment is to give you some initial practice using `Maple`, or a comparable piece of mathematics software. Besides graphing, `Maple` and its peers have powerful symbolic and numerical computation features, a couple of which we will take a look at in later assignments, and which are very useful tools in many applications. *If you would rather use a program other than* `Maple` *with comparable capabilities, you may do so.* A substantial fraction of what `Mathematica`, `Maple`'s main rival among commercial programs, can do is exposed for free on WolframAlpha (`www.wolframalpha.com`). `SageMath`, the most comprehensive open-source alternative to both programs, can be downloaded and used for free (`www.sagemath.org`), although it is not as slick as either of its commercial rivals. Please note that if you use a specialized graphing program such as `Desmos` for this assignment, you will not be penalized, but you will be at a disadvantage in later assignments that require other capabilities of `Maple` or its peers.

As far as using `Maple` is concerned, Trent has a fifty-seat site license, and you can access it on a Trent computer via the "Remote Desktop Access" link in the "IT Services" section of the "Services" tab on MyTrent. The fact that it's only a fifty-seat site license means that if everyone in this course tries at once, the majority will be out of luck ... (So don't leave this assignment to the last minute! :-)

Before starting with `Maple`, please read the handout *A very quick start with* `Maple` and play around with `Maple` a little on your own. It would also likely be useful to skim though *Getting started with* `Maple 10` by Gilberto E. Urroz – read those parts concerned with plotting curves more closely! – and perhaps keep it handy as a reference. You can find links to these documents on Blackboard, as well as the MATH 1110H archive page at `euclid.trentu.ca/math/sb/1110H/` . Note that the archive page also has links to various past assignments involving `Maple` and their solutions, at least some of which you may be able to use as models. `Maple`'s help facility may also come in handy, especially when trying to make out the intricacies of what the `plot` command and its options and variations do. Make use of each other and the instructor, too! Don't forget that while you may work together and look stuff up for the assignments, you should write and/or type up what you submit by yourself. *Do not submit a file in one of* `Maple`*'s (or comparable program's) native format, though a printout of one to pdf would be more than acceptable.*

A curve is easy to graph, at least in principle, if it can be described by a function of x in Cartesian coordinates.

1. Use `Maple` to plot the curves defined by $y = 1$ for $-1 \leq x \leq 1$, $y = x^2$ for $-1 \leq x \leq 1$, $y = \sqrt{1 - x^2}$ for $-1 \leq x \leq 1$, and $y = -\frac{1}{3}\sqrt{9 - x^2}$ for $-3 \leq x \leq 3$. Describe each curve informally. [Please submit a pdf of your worksheet(s) if at all possible.] *[2]*

Hint: The first curve is the piece of the horizontal line $y = 1$ for $-1 \leq x \leq 1$. Really not much one can say about it ... :-)

In many cases, a curve is difficult to break up into pieces that are defined by functions of x (or of y) and so is defined implicitly by an equation relating x and y; that is, the curve consists of all points (x, y) such that x and y satisfy the equation. One can, of course, also use implicit definitions to describe curves that can also be defined as the graphs of functions.

2. Use `Maple` to plot the curves implicitly defined by $x^2 + y^2 = 1$, $x^2 + 9y^2 = 9$, and $\left(x^2 + y^2\right)^2 + 4x\left(x^2 + y^2\right) - 4y^2 = 0$, each one for all those (x, y) for which the respective equation holds. Describe each these curves informally. [Please submit a pdf of your worksheet(s) if at all possible.] *[2]*

Hint: `Maple` has a dedicated `implicitplot` command that is part of the `plots` package.

Another way to describe or define a curve in two dimensions is by way of *parametric equations,* $x = f(t)$ and $y = g(t)$, where the x and y coordinates of points on the curve are simultaneously specified by plugging a third variable, called the *parameter* (in this case t), into functions $f(t)$ and $g(t)$. This approach can come in handy for situations where it is impossible to describe all of a curve as the graph of a function of x (or of y) and arises pretty naturally in various physics problems. (Think of specifying, say, the position (x, y) of a moving particle at time t. This is pretty much the context in which Newton invented his version of calculus.)

3. Use `Maple` to plot the parametric curves given by $x = \cos(t)$ and $y = \sin(t)$ for $0 \leq t \leq 2\pi$, by $x = t\cos(t)$ and $y = t\sin(t)$ for $0 \leq t \leq 2\pi$, and $x = 3\cos(t)$ and $y = \sin(t)$ for $\pi \leq t \leq 2\pi$. Describe each of these curves informally. [Please submit a pdf of your worksheet(s).] *[2]*

Polar coordinates are an alternative to the usual two-dimensional Cartesian coordinates. The idea is to locate a point by its distance r from the origin and its direction, which is given by the (counterclockwise) angle θ between the positive x-axis and the line from the origin to the point. Thus, if (r, θ) are the polar coordinates of some point, then its

Cartesian coordinates are given by $x = r\cos(\theta)$ and $y = r\sin(\theta)$. (Note that for purposes of calculus it is usually more convenient to measure angles in radians rather than degrees.) Polar coordinates come in particularly handy when dealing with curves that wind around the origin, since such curves can often be conveniently represented by an equation of the form $r = f(\theta)$ for some function f of θ. If r is negative for a given θ, we interpret that as a distance of $|r|$ in the *opposite* direction, *i.e.* the direction $\theta + \pi$.

4. Use `Maple` to separately plot the curves in polar coordinates given by $r = 1$ for $0 \le \theta \le 2\pi$, by $r = \theta$ for $0 \le r \le 2\pi$, and by $r = 2 - 2\cos(\theta)$ for $0 \le \theta \le 2\pi$. Describe each of these curves informally. [Please submit a printout of your worksheet(s).] *[2]*

5. Some of the curves in problems **1**–**4** are all of or parts of other curves in problems **1**–**4**, obviously with different presentations. Identify all of the curves related in this way that you can. *[2]*

References

1. *A very quick start with* `Maple`, by Stefan Bilaniuk, which can be found (pdf) on Blackboard or the course archive page at: at: `euclid.trentu.ca/math/sb/1110H`
2. *Formula for Success*, by Ruth Brandow, Ellen Dempsey, Marj Tellis, and Lisa Davies, Academic Skills Centre, Trent University, which can be found (pdf) at: `www.trentu.ca/academicskills/documents/ASC_mathematics_000.pdf`
3. *Single Variable Calculus* (Early Transcendentals), by David Guichard, licensed under the Creative Commons Attribution-NonCommercial-ShareAlike License. Available on Blackboard and the course archive page at: `euclid.trentu.ca/math/sb/1110H` May also be downloaded for free from: `communitycalculus.org`
4. *Getting started with Maple 10*, by Gilberto E. Urroz (2005), which can be found (pdf) on Blackboard or the course archive page at: `euclid.trentu.ca/math/sb/1110H` *Thanks to Prof. Urroz for permission to use it!*