

Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Fall 2019

(Typo in question 4 corrected 2019.09.10.)

Assignment #1

Plotting curves with Maple

Due Wednesday, 18 September.

Before attempting the questions below, please read through Chapter 1 of the text for the basics of graphing functions in Cartesian coordinates, and (at least) skim through Chapter 10 for the basics of parametric curves and polar coordinates, respectively. Some basics of graphing in Cartesian coordinates, using trigonometric functions, and so on, can also be found in the Academic Skills pamphlet *Formula for Success*. The basic definitions of how parametric curves and polar coordinates work are given in this assignment for your convenience, but you might want some additional explanations and examples.

As far as using Maple is concerned, please make sure to go to the lab for your section. You should probably also read the handout *A very quick start with Maple* and play around with Maple a little on your own. It might also be useful to skim through *Getting started with Maple 10* by Gilberto E. Urroz – read those parts concerned with plotting curves more closely! – and perhaps keep it handy as a reference. You can find links to these documents on the MATH 1110H web page at euclid.trentu.ca/math/sb/1110H/. Maple's help facility may also come in handy, especially when trying to make out the intricacies of what the `plot` command and its options and variations do. Make use of each other and the instructor, too! Don't forget that while you may work together and look stuff up for the assignments, you should write and/or type up what you submit by yourself.

NOTE. *If you would rather use a comparable program other than Maple, such as Mathematica or SageMath, you may do so, but you're on your own with them.*

A curve is easy to graph, at least in principle, if it can be described by a function of x in Cartesian coordinates.

1. Use Maple to plot the curves defined by $y = \sqrt{1 - x^2}$ for $0 \leq x \leq 1$, $y = -1 + \sqrt{4 - x^2}$ for $-2 \leq x \leq 0$, $y = -1 - \sqrt{8 + 2x - x^2}$ for $-2 \leq x \leq 1$, and $y = -\sqrt{15 + 2x - x^2}$ for $1 \leq x \leq 5$, all on the same graph. [Please submit a printout of your worksheet(s).] Describe each of these curves informally. [2]

Hint: The first curve is the upper-right quarter circle of radius 1 with centre ...

In many cases, a curve is difficult to break up into pieces that are defined by functions of x (or of y) and so is defined implicitly by an equation relating x and y ; that is, the curve consists of all points (x, y) such that x and y satisfy the equation. One can, of course, also use implicit definitions to describe curves that can also be defined as the graphs of functions.

2. Use Maple to separately plot the curves implicitly defined by $\ln(|x - y|) = 0$, $(x - 1)^2 + (y + 1)^2 = 9$, and $y = x \tan(x^2 + y^2)$, each one for all those (x, y) with $-5 \leq x \leq 5$ and $-5 \leq y \leq 5$ for which the equation is holds. [Please submit a printout of your worksheet(s).] Describe each of these curves informally. [2]

Another way to describe or define a curve in two dimensions is by way of *parametric equations*, $x = f(t)$ and $y = g(t)$, where the x and y coordinates of points on the curve are simultaneously specified by plugging a third variable, called the *parameter* (in this case t), into functions $f(t)$ and $g(t)$. This approach can come in handy for situations where it is impossible to describe all of a curve as the graph of a function of x (or of y) and arises pretty naturally in various physics problems. (Think of specifying, say, the position (x, y) of a moving particle at time t .)

3. Use **Maple** to separately plot the parametric curves given by $x = \cos(t)$ and $y = \sin(t)$ for $0 \leq t \leq 2\pi$, and $x = t \cos(t)$ and $y = t \sin(t)$ for $0 \leq t \leq 4\pi$, and $x = 1 + 3 \cos(t)$ and $y = -1 + 3 \sin(t)$ for $\frac{3\pi}{2} \leq t \leq 2\pi$. [Please submit a printout of your worksheet(s).] Describe each of these curves informally. [2]

Polar coordinates are an alternative to the usual two-dimensional Cartesian coordinates. The idea is to locate a point by its distance r from the origin and its direction, which is given by the (counterclockwise) angle θ between the positive x -axis and the line from the origin to the point. Thus, if (r, θ) are the polar coordinates of some point, then its Cartesian coordinates are given by $x = r \cos(\theta)$ and $y = r \sin(\theta)$. (Note that for purposes of calculus it is usually more convenient to measure angles in radians rather than degrees.) Polar coordinates come in particularly handy when dealing with curves that wind around the origin, since such curves can often be conveniently represented by an equation of the form $r = f(\theta)$ for some function f of θ . If r is negative for a given θ , we interpret that as a distance of $|r|$ in the *opposite* direction, *i.e.* the direction $\theta + \pi$.

4. Use **Maple** to separately plot the curves in polar coordinates given by $r = 1$ for $0 \leq \theta \leq \pi$, $r = \sqrt{\theta}$ for $0 \leq \theta \leq 4\pi$, and $r = \frac{1}{\cos(\theta) - \sin(\theta)}$ for all θ with $0 \leq \theta \leq 2\pi$ for which the expression is defined. [Please submit a printout of your worksheet(s).] [2]
5. Some of the curves in problems 1–4 are all of or parts of other curves in problems 1–4, obviously with different presentations. Identify all of the curves related in this way. [2]

REFERENCES

1. *A very quick start with Maple*, by Stefan Bilaniuk, which can be found (pdf) at: euclid.trentu.ca/math/sb/1110H/Fall-2019/MATH1110H-maple-start.pdf
2. *Formula for Success*, by Ruth Brandow, Ellen Dempsey, Marj Tellis, and Lisa Davies, Academic Skills Centre, Trent University, which can be found (pdf) at: www.trentu.ca/academicskills/documents/ASC_mathematics_000.pdf
3. *Single Variable Calculus* (Early Transcendentals), by David Guichard, licensed under the Creative Commons Attribution-NonCommercial-ShareAlike License. May be downloaded for free from: communitycalculus.org
4. *Getting started with Maple 10*, by Gilberto E. Urroz (2005), which can be found (pdf) at: euclid.trentu.ca/math/sb/1110H/Fall-2019/GettingStartedMaple10.pdf