Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
Trent University, Fall 2018

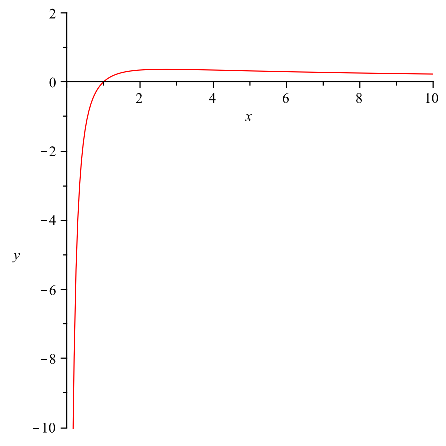
Solutions to Assignment #3
More `Maple`

If you haven't already done so for Assignment #1, you should at least skim through the handout *A very quick start with Maple* and *Getting started with Maple 10*, by Gilberto E. Urroz. For this assignment, you might also profit from checking out *A survey of mathematical applications using Maple 10*, also by Prof. Urroz. As always, you should exploit `Maple`'s own help and tutorials, and this course's *Maple* labs, as necessary. Remember also that you may use other software with similar capabilities instead of `Maple`, such as `Mathematica` or `SageMath`, but it will be your responsibility to learn how to use them to do this assignment if you choose to do so.

1. Use `Maple` to graph $y = \frac{\ln(x)}{x}$ for $0 < x < 10$ and $-10 < y < 2$. Based on this graph, what would you expect $\lim_{x \to \infty} \frac{\ln(x)}{x}$ to be? *[1]*

Solution. Here is the graph:

```
> plot(ln(x)/x, x = 0..10, y = -10..2)
```



The graph suggests that $y = \frac{\ln(x)}{x}$ approaches 0 as $x \to \infty$, *i.e.* that $\lim_{x \to \infty} \frac{\ln(x)}{x} = 0$. ■

2. Use `Maple` actually evaluate $\lim_{x \to \infty} \frac{\ln(x)}{x}$. *[1]*

Solution. Not much to it:

```
> limit(ln(x)/x, infinity)
```

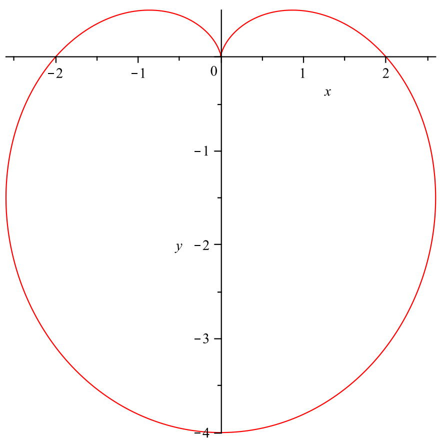
$$0$$

That's all, folks! ■

3. Use `Maple` to graph the curve defined by $\left(x^2+y^2+2y\right)^2 = 4\left(x^2+y^2\right)$. *[2]*

Solution. To use the `implicitplot` command in worksheet mode, you need to invoke the `plots` package:

```
> with(plots);
> implicitplot((x^2+y^2+2*y)^2 = 4*(x^2+y^2), x = -3..3, y = -5..2,
   gridrefine=4)
```



The bounds for x and y were chosen after a bit of trial and error to show the entire cardioid; the `gridrefine=4` option was added after the plot initially turned out a little jagged. ■

4. Use `Maple` to find all the points where $y = x$ intersects $\left(x^2+y^2+2y\right)^2 = 4\left(x^2+y^2\right)$. Use it to find the coordinates of these points both exactly* and as decimals with at least 10 digits of accuracy. *[4]*

Solution. Here's a screenshot of your instructor's attempts to use `Maple` properly,. followed by getting `Maple` to do the job by doing the initial substitution by hand:

> $solve\left(\left\{y=x,\ \left(x^2+y^2+2\cdot y\right)^2=4\cdot\left(x^2+y^2\right)\right\},\ \{x,y\}\right)$

$\{x=0, y=0\},\ \{x=0, y=0\},\ \{x=RootOf(\_Z^2+2\_Z-1), y=RootOf(\_Z^2+2\_Z-1)\}$ **(3)**

> $fsolve\left(\left\{y=x,\ \left(x^2+y^2+2\cdot y\right)^2=4\cdot\left(x^2+y^2\right)\right\},\ \{x,y\}\right)$

$f(\{x=0, y=0\},\ \{x=0, y=0\},\ \{x=RootOf(\_Z^2+2\_Z-1), y=RootOf(\_Z^2+2\_Z-1)\})$ **(4)**

> $solve\left(\left(x^2+x^2+2\cdot x\right)^2=4\cdot\left(x^2+x^2\right),\ x\right)$

$0, 0, \sqrt{2}-1, -1-\sqrt{2}$ **(5)**

> $fsolve\left(\left(x^2+x^2+2\cdot x\right)^2=4\cdot\left(x^2+x^2\right),\ x\right)$

$-2.414213562, 0., 0., 0.4142135624$ **(6)**

>

Since the points of intersection are on $y = x$, you can get the y-coordinates of the points by simply repeating the x-coordinates. ■

---

* For example, $x = \dfrac{-1+\sqrt{5}}{2}$ and $x = \dfrac{-1-\sqrt{5}}{2}$ are the exact solutions of $x^2+x-1=0$.

5. Find the (exact!) coordinates of all the points (x,y) where the line $y = x$ intersects the curve $\left(x^2+y^2+2y\right)^2 = 4\left(x^2+y^2\right)$ yourself. Show all your work! *[2]*

SOLUTION. First, since $y = x$ for the points of intersection, we substitute x for y in $\left(x^2+y^2+2y\right)^2 = 4\left(x^2+y^2\right)$ to obtain $\left(x^2+x^2+2x\right)^2 = 4\left(x^2+x^2\right)$. We then work on solving this equation:

$$\begin{aligned}\left(x^2+x^2+2x\right)^2 = 4\left(x^2+x^2\right) &\iff \left(2x^2+2x\right)^2 = 4\cdot 2x^2 \iff 2^2\left(x^2+x\right)^2 = 8x^2 \\ &\iff 4\left(x^4+2x^3+x^2\right) = 8x^2 \iff x^4+2x^3+x^2 = 2x^2 \\ &\iff x^4+2x^3-x^2 = 0 \iff x^2\left(x^2+2x-1\right) = 0\end{aligned}$$

Thus $\left(x^2+x^2+2x\right)^2 = 4\left(x^2+x^2\right)$ exactly when $x^2\left(x^2+2x-1\right) = 0$, that is, exactly when $x^2 = 0$ or $x^2+2x-1 = 0$. $x^2 = 0$ exactly when $x = 0$, and the quadratic formula tells us when $x^2+2x-1 = 0$:

$$x = \frac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot(-1)}}{2\cdot 1} = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

Thus the points of intersection have x-coordinates $x = 0$, $x = -1+\sqrt{2}$, and $x = -1-\sqrt{2}$, respectively. Since they are on the line $y = x$, the y-coordinates are the same as the x-coordinates in each case, so the three points of intersection are, from left to right:

$$\left(-1-\sqrt{2}, -1-\sqrt{2}\right), \quad (0,0), \quad \text{and} \quad \left(-1+\sqrt{2}, -1+\sqrt{2}\right) \qquad \blacksquare$$