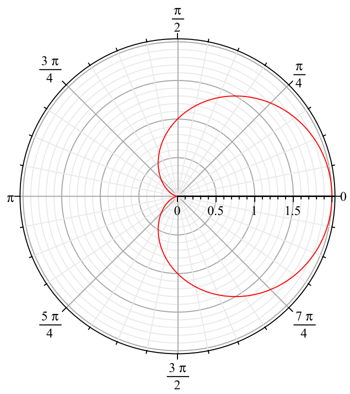
Mathematics 1101Y – Calculus I: Functions and calculus of one variable
TRENT UNIVERSITY, 2013–2014

Assignment #♡
♡ Curves
Due on Monday, 10 March, 2014.

A *cardioid* is one of a family of heart-shaped curves; the polar curve $r = 1 + \cos(\theta)$, for $0 \le \theta \le 2\pi$, from problem **4** on Assignment #1 is a common example of one:



In this assignment we will consider the very similar cardioid given

- algebraically in Cartesian coordinates by $\left(x^2 + y^2 + 2y\right)^2 = 4\left(x^2 + y^2\right)$;
- parametrically in Cartesian coordinates by $x = \dfrac{8t}{\left(t^2+1\right)^2}$ and $y = \dfrac{4\left(t^2-1\right)}{\left(t^2+1\right)^2}$, for $-\infty < t < \infty$ [technically, this parametrization omits the point $(0,0)$]; and
- in polar coordinates by $r = 2\left(1 - \sin(\theta)\right)$, for $0 \le \theta \le 2\pi$.

1. Plot all three descriptions of the given cardioid. *[1.5]*

2. Pick two of the three descriptions and show that all the points given by one of them are also given by the other. *[2.5]*

3. Find the area of the region enclosed by the given cardioid. *[3]*

Hint: This is most easily done in polar coordinates. You can look up how to compute areas in polar coordinates in §11.4.

4. Find the arc-length of the given cardioid. *[3]*

Hint: You can look up how to compute the arc-length of a curve in the textbook, too: §8.1 for doing so for Cartesian curves, §11.2 for parametric curves, and §11.4 for polar curves.

A little something mathematically poetic for Valentine's Day:

Love and Tensor Algebra

Come, let us hasten to a higher plane,
Where dyads tread the fairy fields of Venn,
Their indices bedecked from one to n,
Commingled in an endless Markov chain!

Come, every frustrum longs to be a cone,
And every vector dreams of matrices.
Hark to the gentle gradient of the breeze:
It whispers of a more ergodic zone.

In Riemann, Hilbert or in Banach space
Let superscripts and subscripts go their ways.
Our asymptotes no longer out of phase,
We shall encounter, counting, face to face.

I'll grant thee random access to my heart,
Thou'lt tell me all the constants of thy love;
And so we two shall all love's lemmas prove,
And in our bound partition never part.

For what did Cauchy know, or Christoffel,
Or Fourier, or any Boole or Euler,
Wielding their compasses, their pens and rulers,
Of thy supernal sinusoidal spell?

Cancel me not – for what shall then remain?
Abscissas, some mantissas, modules, modes,
A root or two, a torus and a node:
The inverse of my verse, a null domain.

Ellipse of bliss, converge, O lips divine!
The product of our scalars is defined!
Cyberiad draws nigh, and the skew mind
Cuts capers like a happy haversine.

I see the eigenvalue in thine eye,
I hear the tender tensor in thy sigh.
Bernoulli would have been content to die,
Had he but known such $a^2 cos(2\phi)$!

Stanislaw Lem

(From *The Cyberiad*; translated from the Polish by Michael Kandel.)