Mathematics 1101Y – Calculus I: Functions and calculus of one variable
TRENT UNIVERSITY, 2012–2013

Assignment #γ
Euler's other constant
Due on Friday, 5 April, 2013.

Euler was the first to call the base of the natural exponential and logarithmic functions e, and it is often called the Euler constant now. It's not the only constant named after him, though; the following number is also called the Euler (or the Euler-Mascheroni) constant nowadays:

$$\gamma = \lim_{n\to\infty} \left[\left(\sum_{i=1}^{n} \frac{1}{i} \right) - \ln(n+1) \right]$$

This constant turns up in various odd places; among other things, it has connections with the factorial function, $n! = n(n-1)\cdots 2 \cdot 1$.

1. Show that the limit defining γ exists and that $0 < \gamma < 1$. *[10]*

HINT: Interpret $\left(\sum_{i=1}^{n} \frac{1}{i} \right) - \ln(n+1)$ as an area.