Mathematics 1101Y – Calculus I: Functions and calculus of one variable
Trent University, 2012–2013

Assignment #6
Alternative series
Due on Friday, 5 April, 2013.

Recall from class that the *harmonic series,*

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \cdots,$$

diverges. As we'll soon see, the Alternating Series Test allows us to conclude that the *alternating harmonic series,*

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9} - \cdots,$$

converges. How do other ways of sprinkling minus signs in the harmonic series affect convergence?

1. Determine whether
$$1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{7} - \frac{1}{8} + \frac{1}{9} + \cdots$$
$[+ + -- $ repeated pattern of signs] converges or diverges. *[3]*

2. Determine whether
$$1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9} + \cdots$$
$[+ - + $ repeated pattern of signs] converges or diverges. *[3]*

3. Determine whether
$$1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} - \frac{1}{8} - \frac{1}{9} - \cdots$$
[groups of +s and −s, each double the length of the previous one, alternate] converges or diverges. *[3]*

4. If you sprinkle the +s and −s into the harmonic series at random, so that it is equally likely that you will next encounter a + as a −, will the series converge or not? Why? *[1]*

Bonus. In each of **1** – **3** above, find the actual sum of the series. *[3]*