Mathematics 1101Y – Calculus I: Functions and calculus of one variable
TRENT UNIVERSITY, 2012–2013

Assignment #5
Riemann Sums
Due on Friday, March, 2013.

Recall that definite integrals are defined in terms of the area between the graph of the function being integrated and the x-axis, where area above the x-axis is positive and area below the x-axis is negative. (The Fundamental Theorem of Calculus tells us that we can compute definite integrals by evaluating the antiderivative of the function being integrated at the nedpoints of the interval being integrated over.) The formal definition given in §of the textbook is in terms of (complicated!) limits of Riemann sums, that is, of sums of the areas of rectangles approximating the given area. This definition needs to be pretty complicated to handle various possibilities for functions behaving in complicated ways, especially where discontinuities are concerned. However, for functions that are reasonably well-behaved, *i.e.* continuous except maybe at a very few points, we can get away with simpler definitions. Two of these are:

$$\text{THE RIGHT-HAND RULE:} \quad \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{b-a}{n} f\left(a + i\frac{b-a}{n}\right)$$

$$\text{THE LEFT-HAND RULE:} \quad \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{b-a}{n} f\left(a + (i-1)\frac{b-a}{n}\right)$$

You may recall that we mentioned the Right-Hand Rule in class. There are also Mid-Point, Trapezoidal, and many other such rules, which trade off complexity of definition against quality approximations in each piece of the sum in various ways.

1. Explain the Right- and Left-Hand Rule formulas in your own words and/or pictures. *[2]*

2. Use `Maple` to compute the Right- and Left-Hand Rule sums (the parts after the limits above) for $n = 10$, 20, and 100 respectively for each of the following integrals:

$$\textbf{a.} \int_{-1}^{1} x\,dx \qquad \textbf{b.} \int_{1}^{5} \frac{1}{x}\,dx \qquad \textbf{c.} \int_{0}^{\pi} \sin(x)\,dx$$

How close is each sum to the corresponding integral? What patterns do you discern here? *[8]*