Review

Algebra:

**Polynomials** – An expression constructed from variables and constants, using only the operations of addition, subtraction, multiplication, and non-negative, whole number exponents.

**Degree** – Highest power in a polynomial that appears before a non 0 coefficient

**Root** – value of the input variable that makes the output value = 0 (x-intercept)

Ex:

|  |  |  |
| --- | --- | --- |
| Root3x + 46 = 03x = – 46x = $\frac{-46}{3}$ | Linear Rootax + b = 0x = $\frac{-b}{a}$ | Quadratic Root (Use quadratic formula)ax2 + bx + c = 0 $x= \frac{-b \pm \sqrt{b^{2}- 4ac}}{2a}$ |

**Quadratic Formula Proof**

Factor out a:

x2 + $\frac{b}{a}x$ + $\frac{c}{a}$ = $\frac{0}{a}$ = 0 where a ≠ 0

* Complete the square:
* take half the coefficient of the x-term, square it and add the square to both sides

$ \frac{b}{2a}$ $\left(\frac{b}{2a}\right)^{2}$

 $x^{2}+\frac{b}{a}x+\left(\frac{b}{2a}\right)^{2}=-\frac{c}{a}+\left(\frac{b}{2a}\right)^{2}$

* simplify

$x^{2}+\frac{b}{a}x+\frac{b^{2}}{4a^{2}}=-\frac{c}{a}+\frac{b^{2}}{4a^{2}}$

* factor the left side / find lowest common denominator on right

$\left(x+\frac{b}{2a}\right)\left(x+\frac{b}{2a}\right)=-\frac{c}{a}·\frac{4a}{4a}+\frac{b^{2}}{4a^{2}}$

* simplify

$\left(x+\frac{b}{2a}\right)^{2}=-\frac{4ac}{4a^{2}}+\frac{b^{2}}{4a^{2}}$

* square root both sides

$\sqrt{\left(x+\frac{b}{2a}\right)^{2}}=\pm \sqrt{\frac{b^{2}-4ac}{4a^{2}}}$

* simplify

$x+\frac{b}{2a}=\pm \frac{\sqrt{b^{2}-4ac}}{2a}$ $x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$

**Completing the square**

$a(x+p)^{2}+q$ where p and q are constant

$ax^{2}+bx+xc= a(x+p)^{2}+q$

$x^{2}+\frac{b}{a}x+\frac{c}{a}=(x+p)^{2}+\frac{q}{a}$

$x^{2}+\frac{b}{a}x+\frac{c}{a}=x^{2}+2px+q^{2}+\frac{q}{a}$ for this to be true $\frac{b}{a}=2p$ $\frac{c}{a}=q^{2}+\frac{q}{a}$

incomplete

**Completing the square formula**

$a\left(x+\frac{b}{2a}\right)^{2}+\left[c-\frac{b^{2}}{4a}\right]$

**Vertex of parabola:** $y=ax^{2}+bx+c$

$\left(-\frac{b}{2a}, c-\frac{b^{2}}{4a}\right)$

**Completing the square – Quadratic Formula**

$a\left(x+\frac{b}{2a}\right)^{2}+\left[c-\frac{b^{2}}{4a}\right]$ = 0

$a\left(x+\frac{b}{2a}\right)^{2}=-c+\frac{b^{2}}{4a}$

$\left(x+\frac{b}{2a}\right)^{2}=\frac{-c+\frac{b^{2}}{4a}}{a}=\frac{\frac{b^{2}}{4a}}{a}-\frac{c}{a}=\frac{b^{2}}{4a^{2}}-\frac{c}{a}$

$x+\frac{b}{2a}=\pm \sqrt{\frac{b^{2}}{4a^{2}}-\frac{c}{a}}$

$x=\frac{-b}{2a}\pm \sqrt{\frac{b^{2}}{4a^{2}}-\frac{4a}{4a}+\frac{c}{a}}=\frac{-b}{2a}\pm \sqrt{\frac{b^{2}}{4a^{2}}-\frac{4ac}{4a^{2}}}$

$x=\frac{-b}{2a}\pm \sqrt{\frac{1}{4a^{2}}(b^{2}-4ac) }=\frac{-b}{2a}\pm \sqrt{\frac{1}{4a^{2}} }∙\sqrt{b^{2}-4ac}=$ $\frac{-b}{2a}\pm \frac{1}{2a}∙\sqrt{(b^{2}-4ac)}$

$x=\frac{-b}{2a}\pm \frac{\sqrt{(b^{2}-4ac)}}{2a}=\pm \frac{-b\sqrt{(b^{2}-4ac)}}{2a}$

Examining a formula: $2x^{2}-4x+8$

* **opens up** because coefficient of $x^{2}$ is 2 which is $>0$
* the **y intercept** occurs when $x=0 ∴\left(0,8\right)$

$y=2\left(0\right)^{2}-4\left(0\right)+8$

* x intercept occurs when $y=0$ (use the quadratic formula) $x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$
* if $b^{2}-4ac$ :
	+ $>2$ then there are 2 intercepts
	+ $=0$ then there is 1 intercept
	+ $<0$ then there are 0 intercepts
	+ Therefore there is no **x intercept** as $\left(-4\right)^{2}-4\left(2\right)\left(8\right)=16-64=-48$
* The vertex is at $\left(-\frac{b}{2a}, c-\frac{b^{2}}{4a}\right)=\left(-\frac{\left(-4\right)}{2\left(2\right)}, 8-\frac{\left(-4\right)^{2}}{4\left(2\right)}\right)=\left(1, 6\right)$

Chapter 1

Section 1.1 Functions and Models

A **function** f is a rule that assigns to each element x in a set D exactly one element, called f(x), in a set E.

**Domain:** The set of valid values that can be entered into the function.

**Range:** The set of all possible values of f(x) as x varies throughout the domain.

**Independent variable:** A symbol that represents an arbitrary number in the domain of a function.

**Dependent variable:** A symbol that represents a number in the range of a function.

**Four ways to represent a function:**

* Verbally (in words)
* Numerically (table of values)
* Visually (a graph)
* Algebraically (explicit formula)

**Arrow Diagram:**

a

f(x)

f(a)

x

**Vertical Line Test:**

A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

**Piecewise Defined Functions:**

Functions that are defined by different formulas in different parts of their domain

**Symmetry**

Even Function: $f\left(-x\right)=f\left(x\right)$

* Is symmetrical along the y-axis

Odd Function: $f\left(-x\right)=-f\left(x\right)$

* With half the graph you can rotate it $180°$ about the origin to obtain the other half

**Increasing and Decreasing Functions**

A function is called increasing on an interval I if:

$f\left(x\_{1}\right)<f\left(x\_{2}\right)$ whenever $x\_{1}< x\_{2}$ in I

A function is called decreasing on I if:

$f\left(x\_{1}\right)>f\left(x\_{2}\right)$ whenever $x\_{1}< x\_{2}$ in I

Section 1.2 Mathematical Models a Catalog of Essential Functions

Mathematical Models:

* is a description often given as a function or equation of a real-world phenomenon
* they help us to understand the phenomenon and perhaps make predictions about future behaviour
* is never a completely accurate representation of the physical situation but an **idealization**
* a good model simplifies reality enough to allow us to use mathematical calculations but is accurate enough to provide valuable conclusions

Interpret

Solve

Formulate

Real-world predictions

Mathematical Model

Mathematical Conclusions

Real-world problem

Test

|  |  |
| --- | --- |
| **Type** | **Description** |
| **Polynomial**Domain: $R=\left(-\infty , \infty \right) $If the leading coefficient $a\_{n}\ne 0$ then the degree of is n. | $P\left(x\right)=a\_{n}x^{n}+a\_{n-1}x^{n-1}+ · · · +a\_{2}x^{2}+ a\_{1}x+a\_{0}$ Where $n\geq 0$ and the numbers $a\_{n}$ to $a\_{0}$ are constants called coefficients. |
| **Linear**Interpolation: estimate value between observed valuesExtrapolation: prediction of value outside of observation | **Polynomial of the 1st degree** $(a\_{1}\ne 0)$$P\left(x\right)=a\_{1}x+a\_{0}$ Slope intercept form: $f\left(x\right)=mx+b$ * grow at constant rate
* use the empirical model for lawless models
 |
| **Quadratic**Forms a parabola derived from $y=ax^{2}$* opens up if $a\_{2}$ > 0
* opens down if $a\_{2}$ < 0
 | **Polynomial of the 2nd degree** $(a\_{2} or a\ne 0)$$P\left(x\right)=a\_{2}x^{2}+a\_{1}x+a\_{0}$ **or** $P\left(x\right)=ax^{2}+bx+c$ Quadratic formula: $x= \frac{-b \pm \sqrt{b^{2}- 4ac}}{2a}$  |
| **Cubic** | **Polynomial of the 3rd degree** $(a\_{3} or a\ne 0)$$P\left(x\right)=a\_{3}x^{3}+a\_{2}x^{2}+a\_{1}x+a\_{0}$ **or**$P\left(x\right)=ax^{3}+bx^{2}+cx+d$  |
| **Power**Even $f\left(x\right)$ result in a parabola. | $f\left(x\right)=x^{n}$ $n=1, 2, 3, …$ If n determines whether the function is even or odd. Also as n increases the parabola becomes steeper when $x\geq 1$. |
| **Root**Domain: [0, $\infty $) | $f\left(x\right)=x^{\frac{1}{n}}=\sqrt[n]{x}$  |
| **Reciprocal function**Forms a hyperbola | $f\left(x\right)=\frac{1}{x}=x^{-1}$  |
|  **Rational**Any rational function can be rewritten sum of a polynomial and another rational function with a degree that is less in the numerator. | $f\left(x\right)=\frac{P\left(x\right)}{Q\left(x\right)}$ where $P\left(x\right)$ and $Q\left(x\right)$ are polynomials and $Q\left(x\right)\ne 0$ $\frac{x^{2}+3x+2}{x-15}$ using long division $\frac{\left(x-15\right)\left(x+18\right)+272}{x-15}=\frac{\left(x-15\right)\left(x+18\right)}{x-15}+\frac{272}{x-15}$$=\left(x+18\right)+\frac{272}{x-15}$ Polynomial + another rational function where the degree is less in the numerator. |
| **Algebraic**Any rational function is automatically an algorithm | A function that can be constructed using algebraic operations starting with polynomials |
| **Trigonometric**Identities:* $sin^{2}\left(θ\right)+sin^{2}\left(θ\right)=1$
* $\sin(\left(2θ\right)=2\sin(\left(θ\right))\cos(\left(θ\right)))$
* $\cos(\left(2θ\right)=cos^{2}\left(θ\right)-sin^{2}\left(θ\right))$

$=2cos^{2}\left(θ\right)-1$ $=1-2sin^{2}\left(θ\right)$ * $tan^{2}\left(θ\right)=sec^{2}\left(θ\right)-1$
 | A function that uses radian measure (sine, cosine, tangent, cotangent, secant, cosecant) n = integer$-1\leq sinx\geq 1$ $\left|sinx\right|\leq 1$ $sinx=0 when x=nπ$$-1\leq cosx\geq 1$ $\left|cosx\right|\leq 1$$\cos(\left(θ\right))=\frac{b}{c}$ $\sin(\left(θ\right))=\frac{a}{c}$ $\tan(\left(θ\right))=\frac{\sin(\left(θ\right))}{\cos(\left(θ\right))}=\frac{a}{b}$$\sec(\left(θ\right)=\frac{1}{\cos(\left(θ\right))})= \frac{c}{b}$  |
| **Exponential**Domain: $\left(-\infty ,\infty \right)$Range: $\left(0,\infty \right)$ | $f\left(x\right)=a^{x}$ (for $a>0$) |
| **Logarithmic** | $f\left(x\right)=log\_{a}\left(x\right)$ (for $a>0$) |
| **Transcendental** | A non-algebraic function. (Trigonometric, exponential and logarithmic functions. |