Mathematics 1101Y – Calculus I: functions and calculus of one variable
TRENT UNIVERSITY, 2010–2011

Test #1
Friday, 19 November, 2010
Time: 50 minutes

Instructions

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator and an aid sheet.

1. Find $\dfrac{dy}{dx}$ in any *three* (3) of **a-e**. *[12 = 3 × 4 each]*

a. $y = x^x$ **b.** $y = \dfrac{1}{1+x^2}$ **c.** $y = \cos\left(\sqrt{x}\right)$ **d.** $y^2 + x = 1$ **e.** $y = x^2 e^{-x}$

2. Do any *two* (2) of **a-d**. *[10 = 2 × 5 each]*

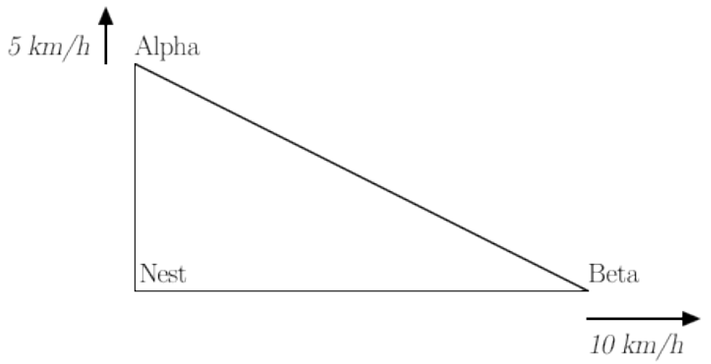
a. Use the limit definition of the derivative to compute $f'(0)$ for $f(x) = x^2 - 3x + \pi$.

b. Suppose $f(x) = \dfrac{x}{\sin(x)}$ for $x \neq 0$. What would $f(0)$ have to be to make $f(x)$ continuous at $a = 0$?

c. Find the equation of the tangent line to $y = x^2$ at the point $(2, 4)$.

d. Use the $\varepsilon - \delta$ definition of limits to verify that $\lim\limits_{x \to 1}(2x + 3) = 5$.

3. Birds Alpha and Beta leave their nest at the same time, with Alpha flying due north at 5 *km/h* and Beta flying due east at 10 *km/h*. How is the area of the triangle formed by their respective positions and the nest changing 1 *h* after their departure? *[8]*



4. Find the domain and all intercepts, maxima and minima, and vertical and horizontal asymptotes of $f(x) = \dfrac{x^2 + 2}{x^2 + 1}$ and sketch its graph based on this information. *[10]*