Mathematics 1101Y – Calculus I: functions and calculus of one variable
TRENT UNIVERSITY, 2010–2011

Solutions to Assignment #3
Some poetical mathematics

Indian mathematicians wrote up much of their work in verse. For example, here is a problem* posed by Bhaskara† (*c.* 1114-1185 A.D.) in a book dedicated to his daughter Lilavati:

> The square root of half the number of bees in a swarm
> Has flown out upon a jasmine bush;
> Eight ninths of the swarm has remained behind;
> And a female bee flies about a male who is buzzing inside a lotus flower;
> In the night, allured by the flower's sweet odour, he went inside it
> And now he is trapped!
> Tell me, most enchanting lady, the number of bees.

For those interested in the history of mathematics, Bhaskara developed a number of techniques based on infinitesimal analysis that anticipated portions of both differential and integral calculus.

1. Restate the problem given above as an equation. *[4]*

SOLUTION. If x is the total number of bees in the swarm, the problem tells us that $\sqrt{\frac{x}{2}}$ of them have flown to the jasmine bush, $\frac{8}{9}x$ have remained behind, and 2 more are in or around the lotus flower. Thus

$$x = \sqrt{\frac{x}{2}} + \frac{8}{9}x + 2$$

is the equation given in the problem. ■

2. Solve the equation you obtained in **1** by hand. *[3]*

SOLUTION. Here goes! We first rearrange the equation to help isolate and get rid of the square root, and then eliminate all the fractions among the coefficients:

$$\begin{aligned} x = \sqrt{\frac{x}{2}} + \frac{8}{9}x + 2 \quad &\Longrightarrow \quad \sqrt{\frac{x}{2}} = \frac{1}{9}x - 2 \\ \Longrightarrow \quad \frac{x}{2} = \left(\frac{1}{9}x - 2\right)^2 &= \frac{1}{81}x^2 - \frac{4}{9}x + 4 \\ \Longrightarrow \quad \frac{1}{81}x^2 - \frac{17}{18}x + 4 = 0 \quad &\Longrightarrow \quad 2x^2 - 153x + 648 = 0 \end{aligned}$$

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* This translation of Bhaskara's problem is given in *The Heritage of Thales*, by W.S. Anglin & J. Lambeck, Springer Verlag, New York, 1995, ISBN 0-387-94544-X, p. 113.

† There was another important Indian mathematician also named Bhaskara (*c.* 600-680 A.D.), just to confuse the issue.

... and then solve the resulting quadratic equation using the quadratic formula:

$$\begin{aligned} x &= \frac{-(-153) \pm \sqrt{(-153)^2 - 4 \cdot 2 \cdot 648}}{2 \cdot 2} \\ &= \frac{153 \pm \sqrt{23409 - 5184}}{4} \\ &= \frac{153 \pm \sqrt{18225}}{4} = \frac{153 \pm 135}{4} \\ &= \frac{288}{4} \quad \text{or} \quad \frac{18}{4} \\ &= 72 \quad \text{or} \quad \frac{9}{2} \end{aligned}$$

Thus x, the number of bees in the swarm, is either 72 or $\frac{9}{2}$. Bhaskara didn't allow for fractional bees ... [But see the bonus problem!] ■

3. Solve the equation you obtained in **1** using `Maple`. *[3]*

SOLUTION. The `Maple` command

```
> solve(x=sqrt(x/2)+8*x/9+2,x);
```

gives the result

$$72$$

That is $x = 72$ is the solution to the equation described by Bhaskara.

On the other hand, the `Maple` command

```
> solve(2*x^2-153*x+648-0,x);
```

gives the result:

$$\frac{9}{2}, 72$$

That is $x = \frac{9}{2}$ and $x = 72$ are the two solutions of the quadratic equation we obtained and then solved in **2**.

Why doesn't `Maple` give $\frac{9}{2}$ as a solution of the original equation? Beats me! ■

Bonus. What does Bhaskara's problem have to do with a *Monty Python* sketch? *[1]*

SOLUTION. The 1972 album *Monty Python's Previous Record* from the British comedy group *Monty Python* includes the song *Eric the Half-a-bee*, where it concludes a variant of their classic *Fish License* sketch. You can find the song on YouTube at:

`http://www.youtube.com/watch?v=MlrsqGal64w`

The lyrics provide a clue that some of the group had been philosophy majors ... ■