Mathematics 1101Y – Calculus I: functions and calculus of one variable
Trent University, 2010–2011

Solutions to Assignment #2
Plotting in Maple and some parametric curves

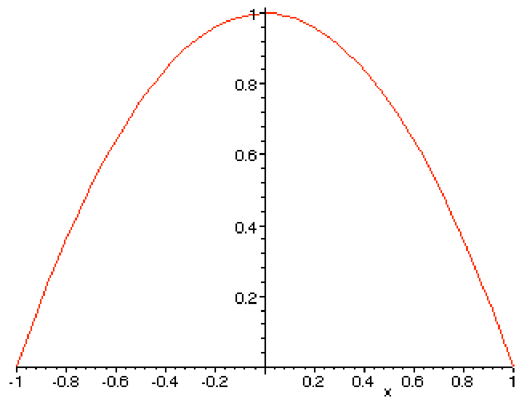
Please see Assignment #2 for the description of Lissajous curves.

1. Use `Maple` to plot the curves $y = 1 - x^2$, $-1 \le x \le 1$, and $x = 1 - y^2$, $-1 \le y \le 1$. Please submit a printout of your worksheet(s) as your solution. *[2]*

Solution. For the first, the `Maple` command

```
> plot(1-x^2,x=-1..1);
```

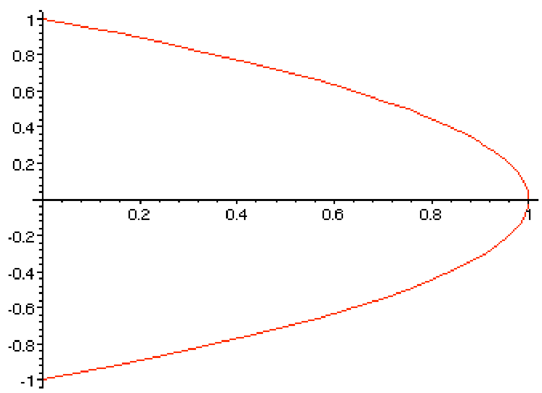
gives:



For the second, the easiest way to get `Maple` to graph x as a function of y is to express it parametrically: $x = 1 - t^2$, $y = t$, $-1 \le t \le 1$. The `Maple` command

```
> plot([1-t^2,t,t=-1..1]);
```

gives:



■

2. Use `Maple` to plot the Lissajous curves for the following combinations of a and b,

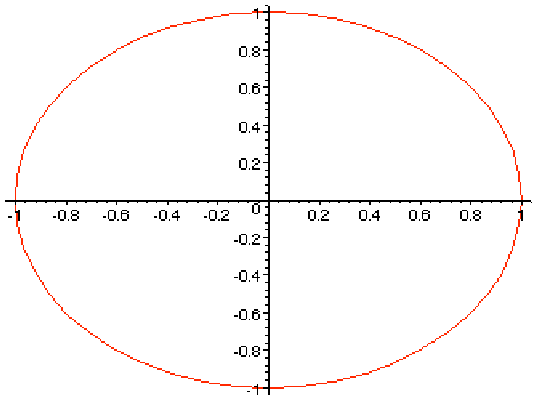
$$\begin{array}{ccccc} a & 1 & 2 & 3 & 4 \\ b & 1 & 1 & 2 & 2 \end{array}.$$

Please submit a printout of your worksheet(s) as your solution. *[4]*

SOLUTION. For $a = 1$, $b = 1$, the `Maple` command

```
> plot([cos(t),sin(t),t=0..2*Pi]);
```

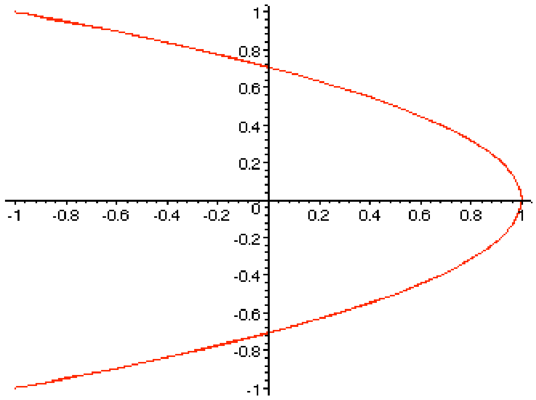
gives:



For $a = 2$, $b = 1$, the `Maple` command

```
> plot([cos(2*t),sin(t),t=0..2*Pi]);
```

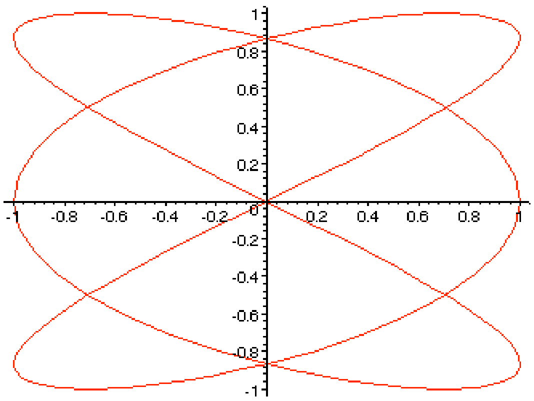
gives:



For $a = 3$, $b = 2$, the `Maple` command

> `plot([cos(3*t),sin(2*t),t=0..2*Pi]);`

gives:

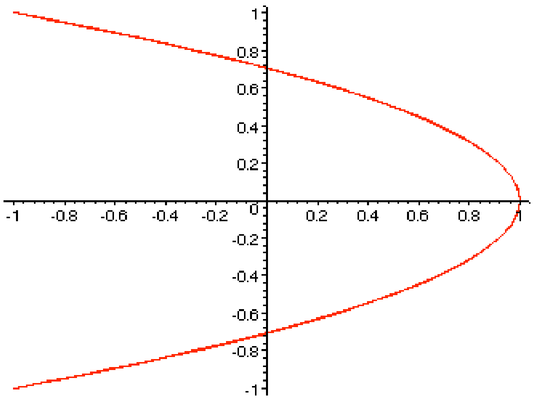


This is the kind of picture people usually have in mind when thinking of Lissajous curves.

Finally, for $a = 4$, $b = 2$, the `Maple` command

> `plot([cos(4*t),sin(2*t),t=0..2*Pi]);`

gives:



■

3. Which combinations of a and b appear to give the same graphs as one of those you obtained in **1**? *[2]*

SOLUTION. Interpreting "the same" to mean "exactly the same," none of them.

Interpreting "the same" to mean "the same type of," then the Lissajous curves for $a = 2$, $b = 1$, and $a = 4$, $b = 2$, respectively, are pieces of of a parabola similar to $x = 1 - y^2$. (In particular, they have the same tip and orientation.)

Either interpretation would have gotten you full credit, assuming the graphs you got in **1** and **2** actually supported what you said. ■

4. Explain why these combinations do give the same graph as one you obtained in **1**. *[2]*

SOLUTION. Nothing need be said here if you answered "none of them" in **3** ... [Really free marks, if you think about it!]

Otherwise, here is why the Lissajous curve for $a = 2$, $b = 1$, gives (a piece of) a parabola similar to $x = 1 - y^2$. If $x = \cos(2t)$ and $y = \sin(t)$, then, using one form of the double-angle formula for cos, we have:

$$x = \cos(2t) = 1 - 2\sin^2(t) = 1 - 2y^2$$

Note that $x = 1 - 2y^2$ is a parabola with its tip at $(1, 0)$ and opening leftwards, just like the parabola $x = 1 - y^2$ from **1**.

The Lissajous curve for $a = 4$, $b = 2$, is (the same) part of the same parabola as the Lissajous curve for $a = 2$, $b = 1$. If $x = \cos(4t)$ and $y = \sin(2t)$, then, using the same double-angle formula for cos, we have:

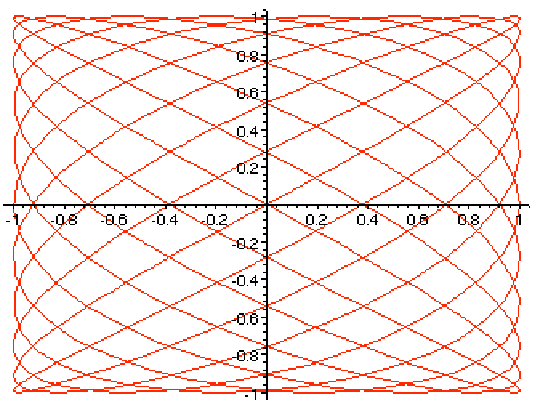
$$x = \cos(4t) = 1 - 2\sin^2(2t) = 1 - 2y^2$$

Note that $4t = 2 \cdot 2t$. ■

Just for fun, for $a = 11$, $b = 8$, the `Maple` command

```
> plot([cos(11*t),sin(8*t),t=0..2*Pi]);
```

gives:



I love these pictures!