Mathematics 1101Y – Calculus I: functions and calculus of one variable
Trent University, 2010–2011

Assignment #4π
The Gamma function
Due on Friday, 8 April, 2011.

The Gamma function can be defined as follows for $x > 0$:

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\, dt$$

There are other ways to define $\Gamma(x)$, but this one is probably the easiest to handle with the usual tools of first-year calculus. For some of the other ways, as well as a some information about the function's relationships to various other functions, check out the *Wikipedia* article about the Gamma function at: `http://en.wikipedia.org/wiki/Gamma_function` Though we won't verify it, $\Gamma(x)$ is differentiable and integrable wherever it is defined. It turns up with some frequency in various parts of applied mathematics.

1. Verify that the above definition of $\Gamma(x)$ does make sense for all $x > 0$, *i.e.* that the improper integral actually converges. *[5]*
2. Compute $\Gamma(1)$. *[1]*
3. Show that $\Gamma(x+1) = x\Gamma(x)$ for all $x > 0$. *[3]*
4. Verify that $\Gamma(n+1) = n!$ for every integer $n \geq 0$. *[1]*

Bonus. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. *[2]*