Math 1100 — Calculus, Quiz #5A — 2009-11-02

Differentiate the following functions:

(30) 1. $h(x) = \sqrt[3]{x} \cdot \sin(x)$.

Solution: $h(x) = f(x) \cdot g(x)$ where $f(x) = \sqrt[3]{x}$ and $g(x) = \sin(x)$. We have $f'(x) = \dfrac{1}{3x^{2/3}}$ and $g'(x) = \cos(x)$. Thus, the Leibniz product rule says:

$$h'(x) \quad = \quad f'(x)g(x) + f(x)g'(x) \quad = \quad \boxed{\frac{\sin(x)}{3x^{2/3}} \; + \; \sqrt[3]{x}\cos(x).}$$

□

(35) 2. $h(x) = \dfrac{1+\sin(x)}{x+\cos(x)}$.

Solution: $h(x) = \dfrac{f(x)}{g(x)}$, where $f(x) = 1 + \sin(x)$ and $g(x) = x + \cos(x)$. We have $f'(x) = \cos(x)$ and $g'(x) = 1 - \sin(x)$. Thus, the Quotient Rule says

$$\begin{aligned} h'(x) \quad &= \quad \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \quad = \quad \frac{\cos(x)\cdot(x+\cos(x)) - (1+\sin(x))\cdot(1-\sin(x))}{(x+\cos(x))^2} \\ &= \quad \frac{x\cos(x) + \cos(x)^2 - 1 + \sin(x)^2}{(x+\cos(x))^2} \\ &\underset{(*)}{=} \quad \frac{x\cos(x) + 1 - 1}{(x+\cos(x))^2} \quad = \quad \boxed{\frac{x\cos(x)}{(x+\cos(x))^2}.} \end{aligned}$$

here, $(*)$ is because $\sin(x)^2 + \cos(x)^2 = 1$ by Pythagoras. □

(35) 3. $h(x) = \sqrt{1 + \exp(x^2)}$.

Solution: $h(x) = f \circ g(x)$ where $f(y) = \sqrt{y}$ and $g(x) = 1 + \exp(x^2)$. We have $f'(y) = \dfrac{1}{2\sqrt{y}}$ and $g'(x) = \exp(x^2) \cdot 2x$ (by the Chain rule). Thus, a second application of the Chain rule yields

$$h'(x) \quad = \quad f'(g(x)) \cdot g'(x) \quad = \quad \boxed{\frac{\exp(x^2)\cdot 2x}{2\sqrt{1+\exp(x^2)}}.}$$

□