Math 1100 — Calculus, Quiz #4B — 2009-10-22

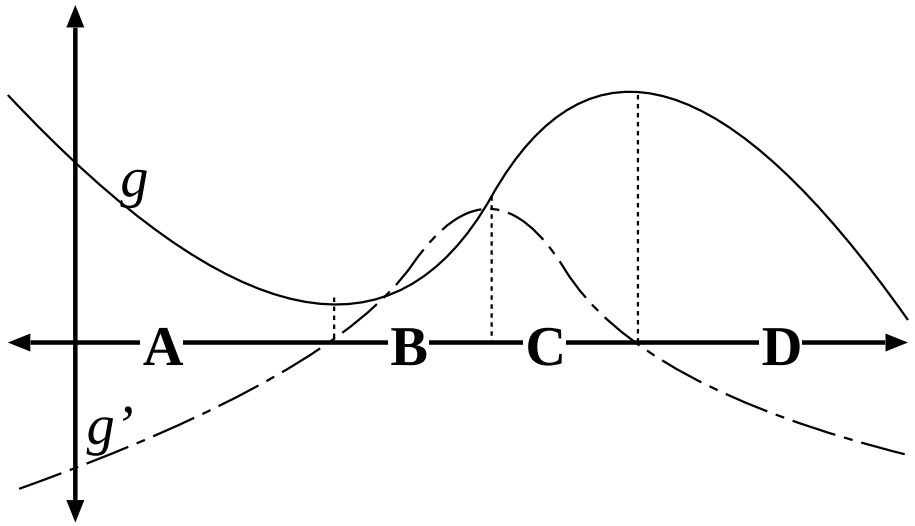
(60) 1. Let $f(x) := 4x^2 - 3x + 1$. Compute the derivative $f'(x)$ using the 'limit' definition of derivative. (Do *not* just apply the 'power rule' to get the answer).

Solution:

$$\begin{aligned} f'(x) &:= \lim_{\epsilon\to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon} = \lim_{\epsilon\to 0} \frac{(4(x+\epsilon)^2 - 3(x+\epsilon) + 1) - (4x^2 - 3x + 1)}{\epsilon} \\ &= \lim_{\epsilon\to 0} \frac{4x^2 + 8x\epsilon + 4\epsilon^2 - 3x - 3\epsilon + 1 - 4x^2 + 3x - 1}{\epsilon} \\ &= \lim_{\epsilon\to 0} \frac{8x\epsilon + 4\epsilon^2 - 3\epsilon}{\epsilon} \\ &= \lim_{\epsilon\to 0} (8x + 4\epsilon - 3) = \boxed{8x - 3.} \end{aligned}$$

□

(40) 2. Here is the graph of the function g. Sketch the graph of its derivative g'. In your sketch, divide the real line into intervals corresponding to regions where g is increasing, decreasing, etc. and relate this to corresponding properties of g'.



Solution: In intervals **A** and **D**, the function g is decreasing and g' is negative.

In interval **B** and **C**, the function g is increasing and g' is positive.

(Bonus) In intervals **A** and **B**, the function g is curving up and g' is increasing.

In intervals **C** and **D**, the function g is curving down and g' is decreasing. □