

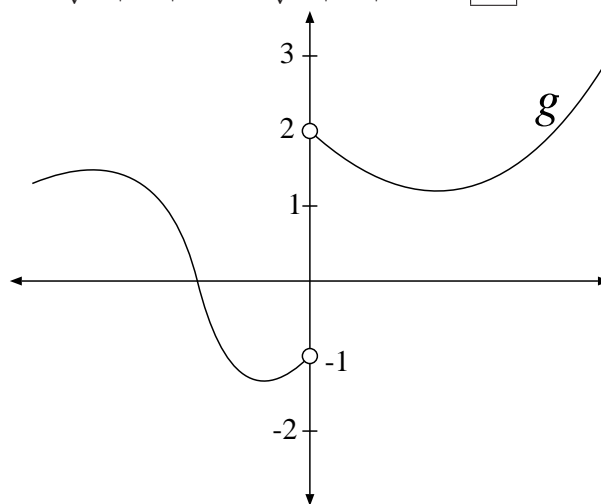
Math 1100 — Calculus, Quiz #2A — 2009-09-28

- (40) 1. Let $f(x) := \frac{\sqrt{1+x}-1}{x}$. Use the ‘limit laws’ from section 2.3 in the book to compute $\lim_{x \rightarrow 0} f(x)$.

Solution: For all $x \neq 0$, we have

$$\begin{aligned} \frac{\sqrt{1+x}-1}{x} &= \frac{\sqrt{1+x}-1}{x} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} = \frac{(\sqrt{1+x})^2 - \sqrt{1+x} + \sqrt{1+x} - 1}{x(\sqrt{1+x}+1)} \\ &= \frac{1+x-1}{x(\sqrt{1+x}+1)} = \frac{x}{x(\sqrt{1+x}+1)} = \frac{1}{\sqrt{1+x}+1}. \end{aligned}$$

Thus, $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x}+1} = \frac{1}{\sqrt{1+0}+1} = \boxed{\frac{1}{2}}$. □



- (30) 2. Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be the function portrayed in the picture. Based on this picture, what are $\lim_{x \nearrow 0} g(x)$ and $\lim_{x \searrow 0} g(x)$?

Solution: $\lim_{x \nearrow 0} g(x) = \boxed{-1}$, while $\lim_{x \searrow 0} g(x) = \boxed{2}$. □

3. Combine your answers from #1 and #2.

- (30) (a) What is $\lim_{x \nearrow 0} (f(x) + g(x))$?
 (b) What is $\lim_{x \searrow 0} f(x) \cdot g(x)$?

Solution: (a) $\lim_{x \nearrow 0} (f(x) + g(x)) = \lim_{x \nearrow 0} f(x) + \lim_{x \nearrow 0} g(x) = \frac{1}{2} + -1 = \boxed{-\frac{1}{2}}$.

(b) $\lim_{x \searrow 0} f(x) \cdot g(x) = \left(\lim_{x \searrow 0} f(x) \right) \cdot \left(\lim_{x \searrow 0} g(x) \right) = \frac{1}{2} \cdot (2) = \boxed{1}$. □