

Math 1100 — Calculus, Quiz #1B — 2009-09-21

Define the functions  $g : \mathbb{R} \rightarrow \mathbb{R}$  and  $f : \mathbb{R} \rightarrow \mathbb{R}$  by  $f(x) = x - 2$  and  $g(x) = 4x$  for all  $x \in \mathbb{R}$ .

1. Find a formula for each of the following functions:

(10)

(a)  $f \circ f$ .

**Solution:**  $f \circ f(x) = x - 2 - 2 = \boxed{x - 4}$ , for all  $x \in \mathbb{R}$ . □

(10)

(b)  $f \circ f \circ f$ .

**Solution:**  $f \circ f \circ f(x) = x - 2 - 2 - 2 = \boxed{x - 6}$ , for all  $x \in \mathbb{R}$ . □

(10)

(c)  $g \circ g$ .

**Solution:**  $g \circ g(x) = 4 \cdot 4 \cdot x = \boxed{16x}$ , for all  $x \in \mathbb{R}$ . □

(10)

(d)  $g \circ g \circ g$ .

**Solution:**  $g \circ g \circ g(x) = 4 \cdot 4 \cdot 4 \cdot x = \boxed{64x}$ , for all  $x \in \mathbb{R}$ . □

(10)

(e)  $f \circ g$ .

**Solution:**  $f \circ g(x) = f[g(x)] = f(4x) = \boxed{4x - 2}$ , for all  $x \in \mathbb{R}$ . □

(10)

(f)  $g \circ f$ .

**Solution:**  $g \circ f(x) = g[f(x)] = g(x - 2) = 4(x - 2) = \boxed{4x - 8}$ , for all  $x \in \mathbb{R}$ . □

2. Define the function  $f : (-5, \infty) \rightarrow \mathbb{R}$  by  $f(x) = \ln(x + 5)$  for all  $x \in (-5, \infty)$ .

(10)

(a) Why is  $f(x)$  not well-defined if  $x \leq -5$ ?

**Solution:** If  $x < -5$ , then  $x + 5 < 0$ , so  $\ln(x + 5)$  is not well-defined. □

(20)

(b) Find a formula for the function  $f^{-1}$ .

**Solution:** For any  $x \in (-5, \infty]$  and  $y \in \mathbb{R}_+$ , we have:

$$\begin{aligned} (y = f(x)) &\iff (y = \ln(x + 5)) \iff (e^y = x + 5) \iff (e^y - 5 = x) \\ &\iff \iff (f^{-1}(y) = e^y - 5) \end{aligned}$$

We conclude that  $\boxed{f^{-1}(y) = e^y - 5}$  for all  $y \in \mathbb{R}$ . □

(10)

(c) What is the range of the function  $f^{-1}$ ?

**Solution:** The range of  $f^{-1}$  is the domain of  $f$  —that is, the interval  $\boxed{(-5, \infty)}$ . □