Math 1100 — Calculus, Quiz #12B — 2010-02-04

(35) 1. Compute $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx$.

Solution: Let $u := \sqrt{x} = x^{1/2}$; then $du = \frac{1}{2\sqrt{x}}\, dx$. Thus, the Substition Rule says

$$\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx \;=\; 2\int \frac{\sin(\sqrt{x})}{2\sqrt{x}}\, dx \;=\; 2\int \sin(u)\, du$$
$$= -2\cos(u) + C \;=\; \boxed{-2\cos(\sqrt{x}) + C.}$$

□

(35) 2. Compute $\int x^3 \cdot \sqrt{x^2 - 1}\, dx$. (**Hint:** *Let $y :=$ the stuff under the $\sqrt{\quad}$ symbol.*)

Solution: Let $y := x^2 - 1$; then $dy = 2x\, dx$, and $x^2 = (y + 1)$. Thus,

$$\int x^3 \cdot \sqrt{x^2 - 1}\, dx \;=\; \frac{1}{2}\int x^2 \cdot \sqrt{x^2 - 1} \cdot 2x\, dx \;=\; \frac{1}{2}\int (y + 1) \cdot \sqrt{y}\, dy$$
$$= \frac{1}{2}\int y^{3/2} + y^{1/2}\, dy \;=\; \frac{1}{2}\left(\frac{2y^{5/2}}{5} + \frac{2y^{3/2}}{3}\right) + C$$
$$= \boxed{\frac{(x^2 - 1)^{5/2}}{5} + \frac{(x^2 - 1)^{3/2}}{3} + C.}$$

□

(30) 3. Let $f(x) := \frac{\sin(\sqrt{x})}{\sqrt{x}}$ for all $x > 1$ (from question #1). Let $g(x) := x^2$ for all $x > 1$. You may assume $f(x) \geq g(x)$ for all $x > 1$. Compute the area of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the vertical lines $x = 1$ and $x = 2$.

Solution: The area is given by

$$\int_1^2 f(x) - g(x)\, dx \;=\; \int_1^2 f(x)\, dx - \int_1^2 g(x)\, dx \;=\; \int \frac{\sin(\sqrt{x}))}{\sqrt{x}}\, dx - \int_1^2 x^2\, dx$$
$$= -2\cos(\sqrt{x})\Big|_{x=1}^{x=2} - \frac{x^3}{3}\Big|_{x=1}^{x=2} \;=\; 2\cos(\sqrt{2}) - 2\cos(\sqrt{1}) + \frac{2^3 - 1^3}{3}$$
$$= \boxed{-2\cos(\sqrt{2}) + 2\cos(1) + \frac{7}{3}.}$$

□