Math 1100 — Calculus, Quiz #12A — 2010-02-01

(35) 1. Compute $\int \frac{\sin(\ln(x))}{x}\, dx$.

Solution: Let $u := \ln(x)$; then $du = \frac{1}{x}\, dx$. Thus, the Substition Rule says

$$\int \frac{\sin(\ln(x))}{x}\, dx \quad = \quad \int \sin(u)\, du \quad = \quad -\cos(u) + C \quad = \quad \boxed{-\cos(\ln(x)) + C.}$$

(Common mistakes were: (1) forgetting the minus sign, and (2) forgetting the $+C$. Each of these cost 5 marks.)

□

(35) 2. Compute $\int \frac{x^2}{\sqrt{2e - x}}\, dx$. (**Hint:** *Let $y :=$ the stuff under the $\sqrt{}$ symbol.*)

Solution: Let $y = 2e - x$; then $dy = -dx$, and $x = 2e - y$, so that $x^2 = (2e - y)^2 = 4e^2 - 2ey + y^2$. Thus, the substitution rule says

$$\begin{aligned}
\int \frac{x^2}{\sqrt{2e - x}}\, dx &= -\int \frac{4e^2 - 4ey + y^2}{\sqrt{y}}\, dy \quad = \quad -\int 4e^2 y^{-1/2} - 4ey^{1/2} + y^{3/2}\, dy \\
&= -4e^2 \int y^{-1/2}\, dy \; - \; 4e \int y^{1/2}\, dy \; + \; \int y^{3/2}\, dy \\
&= -4e^2 \cdot 2\, y^{1/2} \; + \; 4e \cdot \frac{2\, y^{3/2}}{3} \; - \; \frac{2y^{5/2}}{5} \; + \; C \\
&= -8e^2 y^{1/2} \; + \; \frac{8e\, y^{3/2}}{3} \; - \; \frac{2y^{5/2}}{5} + C \\
&= \boxed{-8e^2(2e - x)^{1/2} \; + \; \frac{8e\,(2e - x)^{3/2}}{3} \; - \; \frac{2(2e - x)^{5/2}}{5} \; + \; C.}
\end{aligned}$$

□

(30) 3. Let $f(x) := x^2$ for all $x > 0$. Let $g(x) := \frac{\sin(\ln(x))}{x}$ for all $x > 0$ (from question #1). You may assume $f(x) \geq g(x)$ for all $x > 0$. Compute the area of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the vertical lines $x = 1$ and $x = e$.

Solution: The area is given by

$$\begin{aligned}
\int_1^e f(x) - g(x)\, dx &= \int_1^e f(x)\, dx \; - \; \int_1^e g(x)\, dx \quad = \quad \int_1^e x^2\, dx \; - \; \int \frac{\sin(\ln(x))}{x}\, dx \\
&= \frac{x^3}{3}\Bigg|_{x=1}^{x=e} \; + \; \cos(\ln(x))\Bigg|_{x=1}^{x=e} \quad = \quad \cos(\ln(e)) - \cos(\ln(1)) \; + \; \frac{e^3 - 1^3}{3} \\
&= \frac{e^3}{3} - \frac{1}{3} + \cos(1) - \cos(0) \quad = \quad \frac{e^3}{3} - \frac{1}{3} + \cos(1) - 1 \\
&= \boxed{\frac{e^3 - 4}{3} - \cos(1).}
\end{aligned}$$

□