MATH 1101 2009 Midterm Test 1 Solution

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Dr. Bing Zhou Name _____

Instructions. *All answers should be clear and complete. Show all your work. Partial credit will be given only to the part of your work that leads to a correct answer. If you have any question about the meaning of a problem, ask!* (Total 2 pages and 20 points.)

1. (10 points) Find $\frac{dy}{dx}$. (Do not simplify).

   (a) (2) $y = \left(x^3 + 2x + 1\right) \tan x$
   Solution:
   $$\frac{dy}{dx} = \left(3x^2 + 2\right) \tan x + \left(x^3 + 2x + 1\right) \sec^2 x$$
   □

   (b) (3) $y = e^{\sin 3x}$
   Solution:
   $$\frac{dy}{dx} = e^{\sin 3x} \cdot \cos 3x \cdot 3$$
   □

   (c) (5) $x^2 + y^2 = \ln\left(xy^2\right)$
   Solution:
   $$\begin{aligned} 2x + 2y\frac{dy}{dx} &= \frac{1}{xy^2}\left(y^2 + x\,(2y)\,\frac{dy}{dx}\right) \\ 2x + 2y\frac{dy}{dx} &= \frac{1}{x} + \frac{2}{y}\frac{dy}{dx} \\ \left(2y - \frac{2}{y}\right)\frac{dy}{dx} &= \frac{1}{x} - 2x \\ \frac{dy}{dx} &= \frac{\frac{1}{x} - 2x}{2y - \frac{2}{y}}. \end{aligned}$$
   □

2. (3 points) Find the limit
   $$\lim_{x\to\infty} x \tan\left(\frac{3}{x}\right).$$
   Solution:
   $$\begin{aligned} & \lim_{x\to\infty} x \tan\left(\frac{3}{x}\right) \\ = & \lim_{x\to\infty} \frac{\tan\left(\frac{3}{x}\right)}{\frac{1}{x}} \left(\frac{0}{0}\right) \\ = & \lim_{x\to\infty} \frac{\sec^2\left(\frac{3}{x}\right) \cdot 3 \cdot \left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)} \\ = & \lim_{x\to\infty} \frac{\sec^2\left(\frac{3}{x}\right) \cdot 3}{1} = 3, \end{aligned}$$
   since $\sec(0) = 1$. □

3. (7 points) Let $f(x) = \frac{x}{500x-1}$

   (a) Find the domain of f.
   Solution: Let $500x - 1 = 0$. $x = \frac{1}{500}$. The domain is $\left(-\infty, \frac{1}{500}\right) \cup \left(\frac{1}{500}, \infty\right)$.

(b) Find the horizontal and vertical asymptotes if they exist.

$$\begin{aligned}\lim_{x\to\infty}\frac{x}{500x-1} &= \lim_{x\to\infty}\frac{1}{500-\frac{1}{x}}=\frac{1}{500}\\ \lim_{x\to-\infty}\frac{x}{500x-1} &= \frac{1}{500}\end{aligned}$$

$y=\frac{1}{500}$ is a horizontal asymptote.

$$\begin{aligned}\lim_{x\to\left(\frac{1}{500}\right)^{+}}\frac{x}{500x-1} &= \infty\\ \lim_{x\to\left(\frac{1}{500}\right)^{-}}\frac{x}{500x-1} &= -\infty\end{aligned}$$

$x=\frac{1}{500}$ is a vertical asymptote.

(c) Find the intervals of increase or decrease.

$$f'=\frac{500x-1-500x}{(500x-1)^2}=\frac{-1}{(500x-1)^2}<0$$

$f(x)$ is decreasing on $\left(-\infty,\frac{1}{500}\right)\cup\left(\frac{1}{500},\infty\right)$.

(d) Find the intervals of concavity.

$$\begin{aligned}f'' &= \left(-(500x-1)^{-2}\right)'\\ &= 2(500x-1)^{-3}\cdot 500\\ &= \frac{1000}{(500x-1)^3}\end{aligned}$$

$f''>0$ on $\left(\frac{1}{500},\infty\right)$ and $f''<0$ on $\left(-\infty,\frac{1}{500}\right)$. f is concave downward on $\left(-\infty,\frac{1}{500}\right)$ and concave upward on $\left(\frac{1}{500},\infty\right)$. □