MATH 1101Y 2009 Quiz 7 (a)

1. Let $f(x) = 2x^3 - 3x^2 - 12x$.

   (a) (2 pts) Find the intervals of increase or decrease.

   (b) (1 pts) Find the local maximum and minimum values.

   (c) (2 pts) Find the intervals of concavity and the inflection points.

   Solution: $f'(x) = 6x^2 - 6x - 12$.

   Let $f'(x) = 0$. We have

$$\begin{aligned} 6x^2 - 6x - 12 &= 0 \\ x^2 - x - 2 &= 0 \\ (x+1)(x-2) &= 0 \end{aligned}$$

   $f' = 0$ when $x = -1$ or $x = 2$.

   Since

$$\begin{aligned} f'(-2) &= 6(-2)^2 - 6(-2) - 12 \\ &= 24, \end{aligned}$$

$$f'(0) = -12,$$

   and

$$\begin{aligned} f'(3) &= 6(3)^2 - 6(3) - 12 \\ &= 24, \end{aligned}$$

   $f' > 0$ on $(-\infty, -1) \cup (2, \infty)$ and $f' < 0$ on $(-1, 2)$.

   (a) f is increasing on $(-\infty, -1) \cup (2, \infty)$ and decreasing on $(-1, 2)$.

   (b) f has a local maximum at $x = -1$ with value $f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) = 7$. f has a local minimum at $x = 2$ with value $f(2) = 2(2)^3 - 3(2)^2 - 12(2) = -20$.

   $f''(x) = 12x - 6$. Let $f'' = 0$. We have

$$\begin{aligned} 12x - 6 &= 0 \\ x &= \frac{1}{2}. \end{aligned}$$

   Since $f''(0) = -6$ and $f''(1) = 6$, $f'' < 0$ on $\left(-\infty, \frac{1}{2}\right)$ and $f'' > 0$ on $\left(\frac{1}{2}, \infty\right)$. f has an inflection point at $x = \frac{1}{2}$ and $y = 2\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 - 12\left(\frac{1}{2}\right) = -\frac{13}{2}$. □