

MATH 1101Y 2009 Quiz 16 (a)

1. (2 pts) Eliminate the parameter to find a Cartesian equation of the curve.

$$x = \sqrt{t}, y = e^t$$

Solution: Since $t = x^2$, the Cartesian equation of the curve is

$$y = e^{x^2}.$$

□

2. (3 pts) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the equation

$$x = t + t^4, y = t^2 + t^3.$$

Solution:

$$\frac{dx}{dt} = 1 + 4t^3, \frac{dy}{dt} = 2t + 3t^2.$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t + 3t^2}{1 + 4t^3}.$$

$$\begin{aligned} \frac{d}{dt} \left(\frac{dy}{dx} \right) &= \frac{(2 + 6t)(1 + 4t^3) - (2t + 3t^2)12t^2}{(1 + 4t^3)^2} \\ \frac{d^2y}{dx^2} &= \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}} = \frac{(2 + 6t)(1 + 4t^3) - (2t + 3t^2)12t^2}{(1 + 4t^3)^3}. \end{aligned}$$

□