MATH 1101Y 2009 Quiz 15 (b)

1. (3 pts) Find the length of the curve $y = 3 + 2x^{\frac{3}{2}}$, $0 \le x \le 1$.

   Solution: Since
$$y' = 2 \cdot \frac{3}{2} x^{\frac{1}{2}} = 3\sqrt{x},$$
   we have the length of the curve
$$\begin{aligned} L &= \int_0^1 \sqrt{1 + (y')^2} dx \\ &= \int_0^1 \sqrt{1 + \left(3\sqrt{x}\right)^2} dx = \int_0^1 \sqrt{1 + 9x} dx \end{aligned}$$
   Let $u = 1 + 9x$. $du = 9dx$. $x = 0 \to u = 1$. $x = 1 \to u = 10$.
$$\begin{aligned} L &= \int_1^{10} \sqrt{u} \frac{1}{9} du \\ &= \frac{1}{9} \left[\frac{2}{3} u^{\frac{3}{2}} \right]_1^{10} = \frac{2}{27} \left(10^{\frac{3}{2}} - 1 \right). \end{aligned}$$
   □

2. (2 pts) Set up, but do not evaluate, an integral for the area of the surface obtained by rotating the curve $y = \ln(x + 3)$, $0 \le x \le 1$, about (a) the x-axis and (b) the y-axis.

   Solution:

   (a)
$$\begin{aligned} A &= \int_0^1 2\pi y \sqrt{1 + (y')^2} dx \\ &= 2\pi \int_0^1 \ln(x + 3) \sqrt{1 + \left(\frac{1}{x + 3} \right)^2} dx. \end{aligned}$$

   (b)
$$\begin{aligned} A &= \int_0^1 2\pi x \sqrt{1 + (y')^2} dx \\ &= 2\pi \int_0^1 x \sqrt{1 + \left(\frac{1}{x + 3} \right)^2} dx. \end{aligned}$$
   □