MATH 1101Y 2009 Quiz 15 (a)

1. (3 pts) Find the length of the curve $y^2 = (2x+1)^3$, $0 \le x \le 2$.

   Solution: Since

$$y = (2x+1)^{\frac{3}{2}},$$
$$\frac{d}{dx}\left((2x+1)^{\frac{3}{2}}\right)$$

   we have

$$\begin{aligned} y' &= \frac{3}{2}(2x+1)^{\frac{1}{2}}\left(\frac{1}{2}\right) \\ &= 3(2x+1)^{\frac{1}{2}}. \end{aligned}$$

   The length of the curve is

$$\begin{aligned} L &= \int_0^2 \sqrt{1+(y')^2}dx \\ &= \int_0^2 \sqrt{1+9(2x+1)}dx \\ &= \int_0^2 \sqrt{18x+10}dx = \int_0^2 \sqrt{18x+10}dx \end{aligned}$$

   Let $u = 18x+10$. $du = 18dx$. $x = 0 \to u = 10$. $x = 2 \to u = 46$.

$$\begin{aligned} L &= \int_{10}^{46} \sqrt{u}\frac{1}{18}du = \frac{1}{36}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_{10}^{46} \\ &= \frac{1}{54}\left((46)^{\frac{3}{2}} - (10)^{\frac{3}{2}}\right). \end{aligned}$$

$\square$

2. (2 pts) Set up, but do not evaluate, an integral for the area of the surface obtained by rotating the curve $y = e^x$, $0 \le x \le 1$, about (a) the x-axis and (b) the y-axis.

   Solution:

   (a)

$$\begin{aligned} A &= \int_0^1 2\pi y\sqrt{1+(y')^2}dx \\ &= \int_0^1 2\pi e^x\sqrt{1+e^{2x}}dx. \end{aligned}$$

   (b)

$$\begin{aligned} & \int_0^1 2\pi x\sqrt{1+(y')^2}dx \\ = & \int_0^1 2\pi x\sqrt{1+e^{2x}}dx. \end{aligned}$$

$\square$