Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

1. $\int_1^\infty e^{-3x} dx$.

   Solution:

   $$\begin{aligned} \int_1^\infty e^{-3x} dx &= \lim_{b\to\infty} \int_1^b e^{-3x} dx \\ &= \lim_{b\to\infty} \frac{1}{-3} e^{-3x}|_1^b = \lim_{b\to\infty} \frac{1}{-3}\left(e^{-3b} - e^{-3}\right) \\ &= e^{-3}. \end{aligned}$$

   The integral is convergent. $\qquad\qquad\square$

2. $\int_2^\infty \frac{\ln x}{x} dx$.

   Solution:

   $$\begin{aligned} \int_2^\infty \frac{\ln x}{x} dx &= \lim_{b\to\infty} \int_2^b \frac{\ln x}{x} dx \ \left(\text{Let } u = \ln x. \ du = \frac{1}{x} dx.\right) \\ &= \lim_{b\to\infty} \int_{\ln 2}^{\ln b} u\, du = \lim_{b\to\infty} \left[\frac{u^2}{2}\right]_{\ln 2}^{\ln b} \\ &= \lim_{b\to\infty} \left(\frac{(\ln b)^2}{2} - \frac{(\ln 2)^2}{2}\right) = \infty \end{aligned}$$

   The integral is divergent. $\qquad\qquad\square$