

Evaluate the integral.

1. (2.5 pts) $\int x^2 \sin 4x dx$

Solution:

$$\begin{aligned} & \int x^2 \sin 4x dx \\ &= \frac{1}{4} \left(-x^2 \cos 4x - \int (-\cos 4x)(2x) dx \right) \\ &= -\frac{x^2 \cos 4x}{4} + \frac{1}{2} \int x \cos 4x dx \\ &= -\frac{x^2 \cos 4x}{4} + \frac{x \sin 4x}{8} + \frac{\cos 4x}{32} + C. \end{aligned}$$

□

2. (2.5 pts) $\int \tan^3 x \sec^4 x dx$

Solution 1: Let $u = \tan x$. $du = \sec^2 x dx$.

$$\begin{aligned} & \int \tan^3 x \sec^4 x dx \\ &= \int \tan^3 x \sec^2 x \sec^2 x dx = \int \tan^3 x (\tan^2 x + 1) \sec^2 x dx \\ &= \int u^3 (u^2 + 1) du = \int (u^5 + u^3) du \\ &= \frac{u^6}{6} + \frac{u^4}{4} + C = \frac{\tan^6 x}{6} + \frac{\tan^4 x}{4} + C. \end{aligned}$$

Solution 2: Let $u = \sec x$. $du = \tan x \sec x dx$.

$$\begin{aligned} & \int \tan^3 x \sec^4 x dx \\ &= \int \tan^2 x \sec^3 x \tan x \sec x dx = \int (\sec^2 x - 1) \sec^3 x \tan x \sec x dx \\ &= \int (u^2 - 1) u^3 du = \int (u^5 - u^3) du \\ &= \frac{u^6}{6} - \frac{u^4}{4} + C = \frac{\sec^6 x}{6} - \frac{\sec^4 x}{4} + C. \end{aligned}$$

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