MATH 1100 2008 Section A Final Examination
APRIL 16, 2009

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Instructions.   *Please sign your name on this sheet and return it with your booklets. All answers should be clear and complete. Show all your work. If you have any question about the meaning of a problem, ask!* (Total 100 points.)

1. (12 points) Find $\frac{dy}{dx}$: (Do not simplify.)

   (a) $y = \sqrt{\frac{x^2+1}{x^2+3}}$, (b) $y = \int_1^{2x} \frac{1}{1+\sin t} dt$, (c) $x^{\left(e^{2x}\right)}$.

2. (6 points) Find $\frac{d^2y}{dx^2}$:

   (a) $y = x^2 \ln x$, (b) $y = t^2 + 1$, $x = e^t$.

3. (10 points) Find the limits (justify your statements):

   (a) $\lim_{x \to \infty} \frac{(\ln x)^2}{x}$, (b) $\lim_{x \to \infty} \frac{x^{10}-2x^2+1}{4x^{10}+x^2-7}$, (c) $\lim_{x \to 0^+} \left(e^{2x} - 1\right)^x$.

4. (8 points) Let $f(x) = \frac{x^2-1}{x^3}$. Find where $f(x)$ is increasing, decreasing, concave up and concave down. Find all local maximum, local minimum, points of inflection, horizontal, vertical asymptotes if they exist. Justify your statements.

5. (5 points) A point P is moving along the line whose equation is $y = 2x$. How fast is the distance between P and the point $(3,0)$ changing at the instant when P is at $(3,6)$ if x is decreasing at the rate of 2 units/s at that instant?

6. (27 points) Evaluate:

   (a) $\int \frac{e^x dx}{\sqrt{1-e^{2x}}}$, (b) $\int_{-1}^1 \frac{x}{\sqrt{x^4+2x^2+1}} dx$, (c) $\int \frac{1}{\sqrt{x}(x+4)} dx$,

   (d) $\int x^2 \sin x\, dx$, (e) $\int \frac{dx}{x(x^2+1)}$, (f) $\int \tan^3 x \sec^4 x\, dx$.

7. (8.5 points) Choose one part.

   (a) The region bounded by $y = \sqrt{1-x^2}, x = 0$ and $y = 0$ is rotated about $y = -1$. Find the volume of the resulting solid.

   (b) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis.

8. (5 points) Find the (x,y)-coordinate of the points on the curve $r = 4 - \sin\theta$ where the tangent line is horizontal or vertical.

9. (10.5 points) Determine if the series is convergent or divergent. You must justify your answers.

   (a) $\sum_{n=1}^{\infty} \frac{1}{5^n}$, (b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+2}}$, (c) $\sum_{n=1}^{\infty} \left(\frac{n+2}{3n-1}\right)^n$.

10. (8 points) Choose one part.

    (a) Use the ε, δ-definition of limit to prove $\lim_{x \to 3} \left(\frac{2x+3}{x}\right) = 3$.

    (b) Use the limit definition of derivatives to show that if $y = \frac{t+1}{3t+2}$, then $y' = \frac{-1}{(3t+2)^2}$.

    (c) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-2)^n}{n} x^n$.

Have a good summer!