

Mathematics 110 – Calculus of one variable
Trent University 2003-2004

§A QUIZZES

Quiz #1. Friday, 19 September, 2003. [10 minutes]

12:00 Seminar

1. How close does x have to be to 1 in order to guarantee that $\frac{1}{x}$ is within $\frac{1}{10}$ of 1? [10]

13:00 Seminar

1. Find a value of $\delta > 0$ that ensures that $-1 < \sqrt{x} - 4 < 1$ whenever $-\delta < x - 16 < \delta$. [10]

Leftovers

1. Use the $\varepsilon - \delta$ definition of limits to verify that that $\lim_{x \rightarrow 0} 1 = 1$. [10]

Hint: Try any $\delta > 0$ you like ...

Quiz #2. Monday, 29 September, 2003. [10 minutes]

1. Use the $\varepsilon - \delta$ definition of limits to verify that that $\lim_{x \rightarrow 0} \sin^2(x) = 0$. [10]

Hint: You may use the fact that $|\sin(x)| \leq |x|$.

Quiz #3. Friday, 3 October, 2003. [15 minutes]

12:00 Seminar

Evaluate

1. $\lim_{x \rightarrow \infty} \frac{x-2}{x^2-3x+2}$ [5] 2. $\lim_{x \rightarrow 0} \frac{e^x-1}{e^{2x}-1}$ [5]

13:00 Seminar

Evaluate

1. $\lim_{x \rightarrow 1} \frac{x^2+x-2}{x^2+2x-3}$ [5] 2. $\lim_{x \rightarrow \infty} \frac{(x+4)^2}{41x^2+43x+47}$ [5]

Leftovers

1. For what value(s) of the constant c does $\lim_{x \rightarrow 2} (cx+3) = \lim_{t \rightarrow \infty} \frac{ct^2+3+c}{t^2+1}$? [10]

Quiz #4. Friday, 10 October, 2003. [10 minutes]

12:00 Seminar

1. Use the limit definition of the derivative to find $f'(0)$ if $f(x) = (x+1)^3$. [10]

13:00 Seminar

1. Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \frac{1}{x}$. [10]

Leftovers

1. Use the limit definition of the derivative to find $f'(x)$ if $f(x) = x^2 - 3x$. [10]

Quiz #5. Friday, 17 October, 2003. [10 minutes]

12:00 Seminar

Find $\frac{dy}{dx}$ in each of the following:

1. $y = \ln(\sec(x) + \tan(x))$ [3] 2. $e^{xy} = 2$ [3] 3. $y = \frac{x^2 + 4x + 4}{x + 3}$ [4]

13:00 Seminar

Find $\frac{dy}{dx}$ in each of the following:

1. $y = (1 + x^2) \arctan(x)$ [3] 2. $\tan(x + y) = 1$ [3] 3. $y = \frac{e^x + 1}{e^{2x} - 1}$ [4]

Leftovers

Find $\frac{dy}{dx}$ in each of the following:

1. $y = \sqrt{1 - e^{2x}}$ [3] 2. $y = \frac{\tan(x)}{\cos(x)}$ [3] 3. $\ln(x + y) = x$ [4]

Quiz #6. Friday, 31 October, 2003. [15 minutes]

12:00 Seminar

1. A ladder 5 m long rests against a vertical wall. If the top of the ladder slips down at a rate of 1 m/s, how is angle between the bottom of the ladder and the ground changing when the top of the ladder is 4m above the ground? [10]

13:00 Seminar

1. A searchlight is on an island 8 km from the nearest point, call it P , on the mainland (which has a straight shore). The searchlight makes one revolution each minute. How swiftly is the light beam moving along the shore when it is 6 km from and moving towards P ? [10]

Leftovers

1. Kypalo, walking along a straight path at 4 m/s, passes a tree 3 m from the path. How is the distance between Kypalo and the tree changing 1 s after the Kypalo passes the tree? [10]

Quiz #7. Friday, 7 November, 2003. [15 minutes]

1. Find all the intercepts, the maximum, minimum, and inflection points, and the horizontal and vertical asymptotes of the following function:

12:00: $f(x) = \frac{x}{x^2 + 1}$ **13:00:** $g(x) = \frac{x - 1}{x + 1}$ **Leftovers:** $h(x) = \frac{x^2 + 1}{x}$ [10]

Quiz #8. Friday, 21 November, 2003. [12 minutes]

1. Use the Right-hand Rule to compute the area under $y = f(x)$ for $0 \leq x \leq 2$. [10]

12:00: $f(x) = (x + 1)^2 - 1$ **13:00:** $f(x) = x^2 - 4x + 5$

Leftovers: $f(x) = 2x^2 + x + 3$

Hint: You may use the facts that

$$\sum_{i=1}^n i = \frac{n(n+1)}{2} \quad \text{and} \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}.$$

Quiz #9. Friday, 28 November, 2003. [12 minutes]

1. Compute the following integrals. [10]

12:00: **a.** $\int \frac{\sin(x)}{\cos^2(x)} dx$ **b.** $\int \frac{\sqrt{\ln(x)}}{6x} dx$

13:00: **a.** $\int \frac{\sin(\sqrt{x})}{4\sqrt{x}} dx$ **b.** $\int (\sin^2(x) \cos(x) + \cos(x)) dx$

Leftovers: **a.** $\int \frac{4 \arctan^3(x)}{1+x^2} dx$ **b.** $\int (e^x - 1)^2 e^x dx$

Quiz #10. Friday, 5 December, 2003. [10 minutes]

1. Compute the following integral. [10]

12:00: $\int_0^{\pi/4} \tan(x) \ln(\sec(x)) dx$ **13:00:** $\int_0^1 \frac{(x-1)^2}{x^2+1} dx$

Leftovers: $\int_0^1 (x^2+1)^{15} x^3 dx$

Quiz #11. Friday, 9 January, 2004. [12 minutes]

1. Compute the following integral. [10]

12:00: $\int \sin(2x) \sin(x) dx$ **13:00:** $\int (\tan^2(x) - 1) dx$

Leftovers: $\int \sec^{3/2}(x) \tan(x) dx$

Quiz #12. Friday, 16 January, 2004. [15 minutes]

1. Compute the following integral. [10]

12:00: $\int \frac{\sqrt{x^2-4}}{x^2} dx$ **13:00:** $\int_0^2 x \sqrt{4-x^2} dx$

Leftovers: $\int \frac{x^3}{\sqrt{x^2+9}} dx$

Quiz #13. Friday, 23 January, 2004. [15 minutes]

1. Compute the following integral. [10]

$$\mathbf{12:00:} \quad \int \frac{5}{(x-1)(x^2+2x+2)} dx \quad \mathbf{13:00:} \quad \int \frac{4}{(x^2-1)(x-1)} dx$$

$$\mathbf{Leftovers:} \quad \int \frac{x^2-2x+1}{x^2-x-2} dx$$

Quiz #14. Monday, 2 February, 2004. [15 minutes]

Lecture

1. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 0$, and $x = 1$ about the y -axis. [10]

Leftovers

1. Find the volume of the solid obtained by rotating the region bounded by $y = \cos(x)$ and $y = 0$ for $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$ about the y -axis. [10]

Quiz #15. Friday, 6 February, 2004. [15 minutes]

12:00

1. Find the area of the surface obtained by rotating the curve $y = \frac{x^3}{3}$, $0 \leq x \leq 1$, about the x -axis. [10]

13:00

1. Find the arc-length of the curve $y = -\ln(\cos(x))$, $0 \leq x \leq \frac{\pi}{4}$. [10]

Leftovers

1. Find the area of the surface obtained by rotating the curve $y = \ln(x)$, $1 \leq x \leq e$, about the y -axis. [10]