

Mathematics 110 – Calculus of one variable

Trent University 2003-2004

SOLUTIONS TO ASSIGNMENT #3

For the text of *De Omnibus Rebus*, see Assignment #3.

1. Two mathematical questions are posed in the story. Answer both and explain your answers fully. [10]

Bonus. Who wrote this story? What larger work is it a part of? [0.2]

SOLUTION TO THE BONUS PROBLEM. Charles Lutwidge Dodgson, better known by his pseudonym, Lewis Carroll, wrote the story. It is Knot VIII of *A Tangled Tale*, which ran as a magazine serial in the late 19th century. Lewis Carroll is best known nowadays as the author of *Alice's Adventures in Wonderland* and its sequel, *Through the Looking Glass*. ■

NOTE: The entire text of *A Tangled Tale* can be found online at, among other locations:

<http://euclid.trentu.ca/sb/carroll/tangled/title.html>

SOLUTION TO PROBLEM 1. Here are Carroll's own solutions to the problems posed in *De Omnibus Rebus*:

The Pigs

Problem.—Place twenty-four pigs in four sties so that, as you go round and round, you may always find the number in each sty nearer to ten than the number in the last.

Answer.—Place 8 pigs in the first sty, 10 in the second, nothing in the third, and 6 in the fourth: 10 is nearer ten than 8; nothing is nearer ten than 10; 6 is nearer ten than nothing; and 8 is nearer ten than 6.

The Grurmstipths

Problem.—Omnibuses start from a certain point, both ways, every 15 minutes. A traveller, starting on foot along with one of them, meets one in $12\frac{1}{2}$ minutes: when will he be overtaken by one?

Answer.—In $6\frac{1}{4}$ minutes.

Solution.—Let “ a ” be the distance an omnibus goes in 15 minutes, and “ x ” the distance from the starting-point to where the traveller is overtaken. Since the omnibus met is due at the starting-point in $2\frac{1}{2}$ minutes, it goes in that time as far as the traveller walks in $12\frac{1}{2}$; *i.e.* it goes 5 times as fast. Now the overtaking omnibus is “ a ” behind the traveller when he starts, and therefore goes “ $a + x$ ” while he goes “ x .” Hence $a + x = 5x$; *i.e.* $4x = a$, and $x = \frac{a}{4}$. This distance would be traversed by an omnibus in $\frac{15}{4}$ minutes, and therefore

by the traveller in $5 \times \frac{15}{4}$. Hence he is overtaken in $18\frac{3}{4}$ minutes after starting, *i.e.* in $6\frac{1}{4}$ minutes after meeting the omnibus.

Carroll's solution to the grumstipths problem is complete and correct, if a little short on explanation, but his solution to the pigs problem is just a very clever play on words: he gets away with it by using "nothing" in two slightly different ways. Not that he has much choice, because there is no legitimate way to solve the problem:

Suppose the four sties receive, in order, a , b , c , and d pigs each. The distance a number n is from 10 is given by $|n - 10|$; note that this works irrespective of whether n is greater than, less than, or equal to 10. The requirement that the number of pigs in each sty be closer to 10 than the number in the preceding sty as Her Radiancy goes all the way around the courtyard boils down to:

$$|a - 10| < |b - 10| < |c - 10| < |d - 10| < |a - 10|$$

Since this would require that $|a - 10| < |a - 10|$, which is impossible, the requirement cannot be satisfied no matter how the pigs are allocated. ■