Suppose the ends of a string are attached to two fixed points and it hangs under its own weight. (To keep things relatively simple, we'll assume the string is completely flexible, of uniform composition and density, arbitrarily strong, and undisturbed by any forces aside from gravity and the attachment at those two fixed points.) It turns out that the curve $y = f(x)$ formed by such a string must satisfy a differential equation of the form

$$\frac{d^2y}{dx^2} = k\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

where $k > 0$ is a constant. (See Chapter 9 of the text for basic information about differential equations. For the reasons why the curve must satisfy such a differential equation, take some physics ...)

1. Solve the differential equation for $y = f(x)$ if the two fixed points between which the string is suspended are at $(-2, 2)$ and $2, 2)$ and the lowest point of the string is at $(0, 1)$.   *[10]*

Hint: Let $z = \frac{dy}{dx}$. Rewrite the differential equation in terms of z. Solve the new equation for z. Integrate z to get y. Finally, use the points that you know $y = f(x)$ passes through to solve for the constants that appear in y. You may wish to check out §3.9 in the text for information about the hyperbolic functions, which may play a role in this problem.

Find The Longest Path
[To the tune of *For The Longest Time* by Billy Joel.]

Whoa - oh - oh - oh... find the longest path.
Whoa - oh - oh... find the longest path.

If you said P is NP tonight,
There would still be papers left to write.
I have a weakness: I'm addicted to completeness,
And I keep searching for the longest path.

The algorithm I would like to see
Is of polynomial degree.
But it's elusive; nobody has found conclusive
Evidence that we can find a longest path.

["oo-wah" melody]
I... have... been... hard... working for so long.
I swear it's right, and HE marks it wrong!
Some...how... I'll... feel... sorry when it's done.
GPA 2.1 is more than I hoped for.

Garey, Johnson, Karp and other men (*[voice over]* "and women too")
Tried to make it $O(n \log n)$.
Am I a mad fool if I spend my life in grad school
Forever following the longest path?

Whoa - oh - oh - oh... find the longest path.
Whoa - oh - oh... find the longest path.
Whoa - oh - oh... find the longest path.