Mathematics 110 – Calculus of one variable

FINAL EXAMINATION

Trent University, 24 April, 2002

Time: 3 hours *Brought to you by* Stefan Bilaniuk.

Instructions: Show all your work and justify all your conclusions. *If in doubt about something,* **ask!**

Aids: Calculator; $8.5'' \times 11''$ aid sheet or the pamphlet *Formula for Success*; one brain.

Part I. Do all four of **1** – **4**.

1. Find $\dfrac{dy}{dx}$ in any *three* of **a** – **e**. *[9 = 3 × 3 ea.]*

a. $y = x^2\ln(x) - 1$ **b.** $y = \dfrac{\ln(x)}{x^2} + x$ **c.** $\sin^2(y) = e^{-x^2}$

d. $y = \displaystyle\int_x^0 \cos\left(t^2\right)\, dt$ **e.** $y = \sqrt{\sec^2\left(\arctan(x)\right) - 1}$

2. Evaluate any *three* of the integrals **a** – **e**. *[12 = 3 × 4 ea.]*

a. $\displaystyle\int \frac{2x+3}{\sqrt{1-x^2}}\, dx$ **b.** $\displaystyle\int_{-\pi/4}^{\pi/4} \arctan(x)\, dx$ **c.** $\displaystyle\int \cos^2(t)\, dt$

d. $\displaystyle\int_1^\infty \frac{1}{x^3+x}\, dx$ **e.** $\displaystyle\int \frac{1}{x^2+2x+5}\, dx$

3. Evaluate any *three* of the limits **a** – **e**. *[9 = 3 × 3 ea.]*

a. $\displaystyle\lim_{x\to -1} \frac{x^2+2x+1}{x+1}$ **b.** $\displaystyle\lim_{n\to\infty} \frac{2^n}{4^n+\pi}$ **c.** $\displaystyle\lim_{x\to 0} \frac{\arctan(x)}{x}$

d. $\displaystyle\lim_{n\to\infty} \frac{n^2+2n+2}{2n+2}$ **e.** $\displaystyle\lim_{x\to\pi} \frac{\sin(x)}{x-\pi}$

4. Determine whether the given series converges absolutely, converges conditionally, or diverges in any *three* of **a** – **e**. *[12 = 3 × 4 ea.]*

a. $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n\ln\left(n^2\right)}$ **b.** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(n+1)}{n^2+3n+9}$ **c.** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{4n}}{2n+3}$

d. $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$ **e.** $\displaystyle\sum_{n=0}^{\infty} \frac{\arctan(-n)}{5^n}$

Part II. Do *both* of **5** and **6**.

5. Find the domain, all maximum, minimum, and inflection points, and all vertical and horizontal asymptotes of $f(x) = x\ln(x)$, and sketch its graph. *[14]*

6. Consider the region in the first quadrant bounded by $y = \sin(x)$ and $y = \frac{2}{\pi}x$.

a. Sketch the region. *[2]*

b. Sketch the solid obtained by revolving the region about the y-axis. *[3]*

c. Find the volume of the solid. *[7]*

Part III. Do *one* of **7** or **8**.

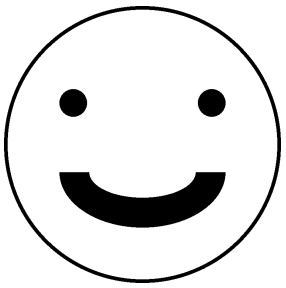
7. Find the MacLaurin series of $\sin(x)$ and determine its radius of convergence. *[12]*

8. Find a function which is equal to $2x + 3x^2 + 8x^3 + 15x^4 + 32x^5 + 63x^6 + 128x^7 + \cdots$, at least when this power series converges. (Note that the coefficient of x^n is 2^n when n is odd and $2^n - 1$ when n is even.) *[12]*

Part IV. Do *one* of **9** or **10**.

9. Use the $\epsilon - \delta$ definition of limits to verify that $\lim_{x \to -3} \frac{x^2 - 9}{x - 3} = 0$. *[10]*

10. A happy face is painted on the surface of a spherical balloon. The face expands as the balloon is inflated at a rate of 10 *litres/sec*. If the distance between the eyes is 10 *cm* at the instant that the diameter of the balloon is 20 *cm*, how is the distance between the eyes changing at the same instant? *[10]*



[Total = 90]

Part V. Bonus!

42_{13} = 6×9. Write a little poem about calculus or mathematics in general. *[2]*

I HOPE YOU'VE HAD FUN IN MATH 110! HAVE A GOOD SUMMER!