Mathematics 1121H – Calculus II
Trent University, Winter 2026

Solutions to Assignment #5
Integrate This![†]
Due on Friday, 13 February.

1. Integrate $\displaystyle\int \frac{1}{1+\sin^2(x)}\,dx$ by hand, showing all the steps. *[10]*

Hint: Believe it or not, you should substitute for a trigonometric function, but not $\sin(x)$.

Solution. We will first use the substitution $w = \tan(x)$, so $dw = \sec^2(x)$, plus some trigonometric manipulation, as follows:

$$\begin{aligned}\int \frac{1}{1+\sin^2(x)}\,dx &= \int \frac{1}{1+\sin^2(x)} \cdot \frac{\sec^2(x)}{\sec^2(x)}\,dx = \int \frac{\sec^2(x)}{\sec^2(x)+\sin^2(x)\sec^2(x)}\,dx \\ &= \int \frac{\sec^2(x)}{\sec^2(x)+\frac{\sin^2(x)}{\cos^2(x)}}\,dx = \int \frac{\sec^2(x)}{1+\tan^2(x)+\tan^2(x)}\,dx \\ &= \int \frac{\sec^2(x)}{1+2\tan^2(x)}\,dx = \int \frac{1}{1+2w^2}\,dw\end{aligned}$$

We follow up with the substitution $u = \sqrt{2}\cdot w$, so $du = \sqrt{2}\cdot dw$, and thus $dw = \dfrac{1}{\sqrt{2}}\,du$. Note that $2w^2 = \left(\sqrt{2}\cdot w\right)^2 = u^2$.

$$\begin{aligned}\int \frac{1}{1+\sin^2(x)}\,dx &= \int \frac{1}{1+2w^2}\,dw = \int \frac{1}{1+u^2}\cdot\frac{1}{\sqrt{2}}\,du = \frac{1}{\sqrt{2}}\int\frac{1}{1+u^2}\,du \\ &= \frac{1}{\sqrt{2}}\cdot\arctan(u)+C = \frac{1}{\sqrt{2}}\arctan\left(\sqrt{2}\cdot w\right)+C \\ &= \frac{1}{\sqrt{2}}\arctan\left(\sqrt{2}\cdot\tan(x)\right)+C \quad \square\end{aligned}$$

Check. We use SageMath to check our work:

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[1]: integral( 1/(1+(sin(x))^2), x )

[1]: 1/2*sqrt(2)*arctan(sqrt(2)*tan(x))
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We got it right! Note that if we had SageMath compute the integral first, the answer gives us a clue as to what substitution to try ...

[†] ... and make my day!