Mathematics 1121H – Calculus II

Trent University, Winter 2026

Assignment #4

Series Business

Due on Friday, 6 February.[*]

Consider the *alternating harmonic series*, $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$. This series *converges*, *i.e.* adds up to a real number, which really means that $\displaystyle\lim_{n\to\infty} \left[\sum_{k=1}^{n} \frac{(-1)^{k+1}}{k}\right]$ exists and is the real number the series adds up to.

1. Use a suitable `for` loop in SageMath to discover the value of n required to ensure that the first two decimal places of $\displaystyle\sum_{k=1}^{n} \frac{(-1)^{k+1}}{k}$ are the same as for the sum of the whole alternating harmonic series. *[2]*

2. What is the sum of the alternating harmonic series? Why? *[2]*

Consider *Gregory's series*, $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k-1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$. This series also converges. (To a different sum than the alternating harmonic series does, mind you.)

3. Use a suitable `for` loop in SageMath to discover the value of n required to ensure that the first two decimal places of $\displaystyle\sum_{k=1}^{n} \frac{(-1)^{k+1}}{2k-1}$ are the same as for the sum of all of Gregory's series. *[4]*

4. What is the sum of Gregory's series? Why? *[2]*

Both the alternating harmonic series and Gregory's series are example of *alternating series*: series that add up non-zero terms a_k that are alternately positive and negative, *i.e.* such that $a_k < 0$ if and only if $a_{k+1} > 0$.

5. Suppose $\displaystyle\sum_{k=1}^{\infty} a_k$ is an alternating series such that $|a_{k+1}| < |a_k|$ for all k and that $\displaystyle\lim_{k\to\infty} |a_k| = 0$. Explain why the series has to converge. [You need not give an actual proof.] *[2]*

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[*] You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper as soon as you can. You may work together, look things up, and use whatever tools you like, so long as you *write up your submission by yourself* and give due credit to your collaborators and any sources and tools you actually used.