Mathematics 1121H – Calculus II
TRENT UNIVERSITY, Winter 2026

Solutions to Assignment #2
Due on Friday, 23 January.

Please read the handout *Right-Hand Rule Riemann Sums* and Section 8.6 of the textbook before tackling this assignment. The subtext to this assignment is to practice things that you should have learned in MATH 1110H/1111H, not counting the Trapezoid and Simpson's Rules.

In all that follows, let $f(x) = xe^{-x}$.

1. Compute $\displaystyle\int_{-1}^{3} f(x)\,dx$ by hand, showing all the principal steps. *[2]*

SOLUTION. We will use integration by parts with $u = x$ and $v' = e^{-x}$, so $u' = 1$ and $v = -e^{-x}$. Note that passing from v' to v is technically an integration in its own right, which can be done via the substitution $w = -x$, so $dw = (-1)\,dx$ and thus $dx = (-1)\,dw$: $\displaystyle\int e^{-x}\,dx = \int e^{w}(-1)\,dw = -e^{w} = -e^{-x}$ In real life, this is trivial enough that it probably isn't worth writing out explicitly.

Anyway, here we go:

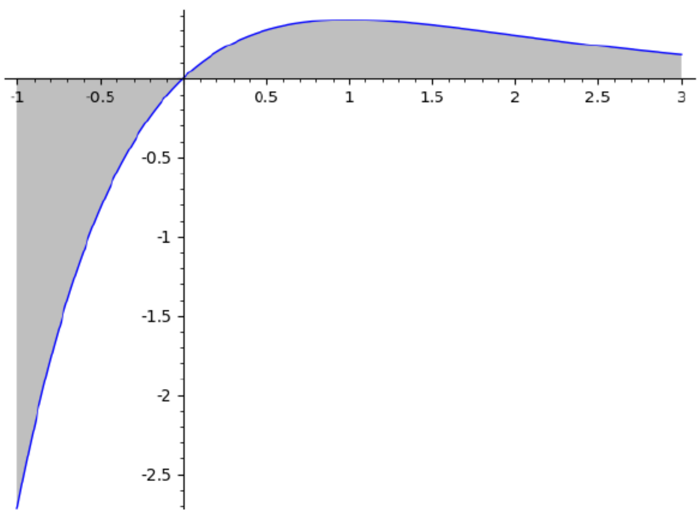
$$\begin{aligned}\int_{-1}^{3} f(x)\,dx &= \int_{-1}^{3} xe^{-x}\,dx = x\left(-e^{-x}\right)\Big|_{-1}^{3} - \int_{-1}^{3} 1\left(-e^{-x}\right)\,dx \\ &= -xe^{-x}\Big|_{-1}^{3} + \int_{-1}^{3} e^{-x}\,dx = \left(-3e^{-3}\right) - \left(-(-1)e^{-(-1)}\right) + \left(-e^{-x}\right)\Big|_{-1}^{3} \\ &= -3e^{-3} - e + \left(-e^{-3}\right) - \left(-e^{-(-1)}\right) = -4e^{-3} - e + e = -4e^{-3} \qquad \square\end{aligned}$$

NOTE. In questions **2**–**5**, please give decimal approximations to at least 4 decimal places as your answers.

Here are the SageMath preliminaries for questions **2**–**5**, with a bonus plot:

```
[1]: f = function('f')(x)
     f(x) = x*e^(-x)
     plot( f(x), -1, 3, fill=True )
```

[1]:



2. Compute $\int_{-1}^{3} f(x)\,dx$ using SageMath. *[2]*

SOLUTION. Here we go:

```
[2]: integral( f(x), x, -1, 3 )
```

```
[2]: -4*e^(-3)
```

```
[3]: N(-4*e^(-3))
```

```
[3]: -0.199148273471456
```

□

Additional SageMath preliminaries for questions **3**–**5**:

```
[4]: var('n')  # The number of equal pieces we'll partition the interval [a,b] into.
     var('i')  # i, j, and k will be used as indices in the sums
     var('j')
     var('k')
     a = -1    # The left endpoint of the interval.
     b = 3     # The right endpoint of the interval.
     R = function('R')(n)
     T = function('T')(n)
     S = function('S')(n)
     # Right-Hand Rule sum
     R(n) = (b-a)/n * sum( f(a+i*(b-a)/n), i, 1, n )
     # Trapezoidal Rule sum
     T(n) = (b-a)/n * ( f(a)/2 + sum( f(a+i*(b-a)/n), i, 1, n-1 ) + f(b)/2 )
     # Simpson's Rule sum
     S(n) = (b-a)/(3*n) * ( f(a) + 4*sum( f(a+(2*j-1)*(b-a)/n), j, 1, n/2 ) + 2*sum(
     ↪f(a+2*k*(b-a)/n), k, 1, n/2-1 ) + f(b) )
```

3. Use SageMath to compute the Right-Hand Rule sum for $\int_{-1}^{3} f(x)\,dx$ for a partition of $[-1,3]$ into **a.** 4, **b.** 16, and **c.** 64 equal pieces, respectively. *[1.5 = 3×0.5 each]*

SOLUTION. Here we go:

```
[5]: N(R(4))
```

```
[5]: 0.787911212748259
```

```
[6]: N(R(16))
```

```
[6]: 0.130531914606309
```

```
[7]: N(R(64))
```

```
[7]: -0.111336326460994
```

□

4. Use SageMath to compute the Trapezoid Rule sum for $\int_{-1}^{3} f(x)\,dx$ for a partition of $[-1,3]$ into **a.** 4, **b.** 16, and **c.** 64 equal pieces, respectively. *[1.5 = 3×0.5 each]*

```
[8]: N(T(4))
```

```
[8]: -0.645910304033059
```

```
[9]: N(T(16))
```

```
[9]: -0.227923464589021
```

```
[10]: N(T(64))
```

□

SOLUTION. Here we go:

5. Use SageMath to compute the Simpson's Rule sum for $\int_{-1}^{3} f(x)\,dx$ for a partition of $[-1,3]$ into **a.** 4, **b.** 16, and **c.** 64 equal pieces, respectively. *[1.5 = 3×0.5 each]*

SOLUTION. Here we go:

```
[11]: N(S(4))
```

```
[11]: -0.250159825039889
```

```
[12]: N(S(16))
```

```
[12]: -0.199381640661294
```

```
[13]: N(S(64))
```

```
[13]: -0.199149194559208
```

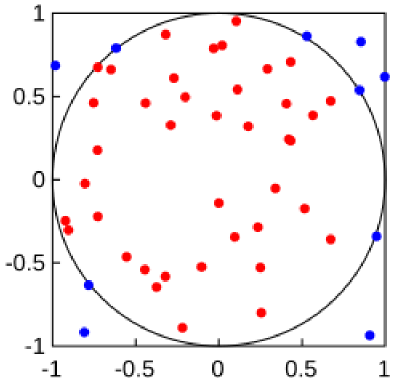
□

6. Find a numerical approximation algorithm or formula for definite integrals besides the three mentioned on this assignment and give a reference to it. *[1.5]*

SOLUTION. One class of numerical algorithms for computing integrals, especially multi-variable ones, are the *Monte Carlo* methods. In these, the area (or volume, or ...) of a finite region is estimated by enclosing it in a rectangle (or a box, or ...), randomly picking a large number points in the rectangle, counting how many of these are in the region, and approximating the area of the region by $\frac{\#\text{ points in region}}{\text{total }\#\text{ of points}} \cdot$ area of rectangle.

For example, this plot illustrates a Monte Carlo calculation of the area of the circle $x^2 + y^2 = 1$, yielding a value of 3.2, not too far off the actual value of π:

This plot was taken from the Wikipedia article about Monte Carlo integration, which you can find at: `en.wikipedia.org/wiki/Monte_Carlo_integration`



□