Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Winter 2024
Final Examination
11:00-14:00 on Saturday, 13 April, in the Gym.

Time: *3 hours.* *Brought to you by* Стефан Біланюк.

Instructions: Do parts **A**, **B**, and **C**, and, if you wish, part **D**. Show all your work and justify all your answers. *If in doubt about something,* **ask!**

Aids: Open book, most any calculator, one head-mounted neural net.

Part A. Do all four (4) of **1**–**4**.

1. Evaluate any *four* (4) of the integrals **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\int_0^\infty \frac{1}{(x+2)^3}\,dx$ **b.** $\displaystyle\int 4xe^{x^2+1}\,dx$ **c.** $\displaystyle\int_0^{\pi/2} \sin^{17}(x)\cos(x)\,dx$

d. $\displaystyle\int \frac{1}{x^2-1}\,dx$ **e.** $\displaystyle\int_1^e \ln(x)\,dx$ **f.** $\displaystyle\int \frac{1}{4-x^2}\,dx$

2. Determine whether the series converges in any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\sum_{n=0}^\infty \frac{n\sqrt{n}}{n^3+1}$ **b.** $\displaystyle\sum_{n=2}^\infty \frac{(-1)^n}{\ln(n^2)}$ **c.** $\displaystyle\sum_{n=0} \frac{n+1}{\pi^n}$

d. $\displaystyle\sum_{n=0}^\infty \frac{3^{n-1}}{(n+1)!}$ **e.** $\displaystyle\sum_{n=1}^\infty \frac{\cos(n^2)}{n^2}$ **f.** $\displaystyle\sum_{n=0}^\infty n^2e^{-n}$

3. Do any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. Find the centroid of the region above $y=0$ and below $y=2$ for $0 \le x \le 2$.

b. Find the arc-length of the curve $y = x + 41$, where $0 \le x \le 4$.

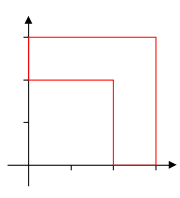
c. Find the sum of the series $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2+n}$.

d. Find the volume of the solid obtained by revolving the region between $y = x-4$ and $y=1$, where $4 \le x \le 5$, about the y-axis.

e. Determine whether the series $\displaystyle\sum_{n=0}^\infty \frac{(-n)^n}{23^n}$ converges or diverges.

f. Find the area of the finite region between $y=x$ and $y=x^4$.

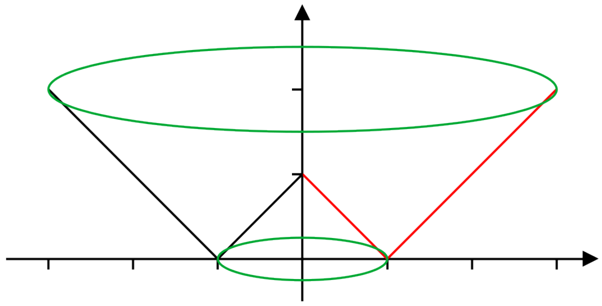
4. Find the centroid of the "bent finger" region below $y=3$ for $0 \le x \le 3$, and above $y=2$ for $0 \le x \le 2$ but above $y=0$ for $2 \le x \le 3$. *[12]*



Parts **B**–**D** *are on page 2.*

Part **A** *is on page 2.*

Part B. Do either *one* (1) of **5** or **6**. *[14]*



5. A solid is obtained by revolving the region below $y = 2$, and above $y = 1 - x$ for $0 \leq x \leq 1$ but above $y = x - 1$ for $1 \leq x \leq 3$, about the y-axis. Find the volume of this solid. *[14]*

6. Find the arc-length of the curve $y = \sqrt{4 - x^2}$, where $0 \leq x \leq 2$,

a. using the arc-length formula and calculus *[10]*, and

b. without using the arc-length formula or calculus. *[4]*

Part C. Do either *one* (1) of **7** or **8**. *[14]*

7. Find the Taylor series at 0 of $f(x) = e^{3x}$

a. using Taylor's formula, *[10]* and

b. without using Taylor's formula, at least directly. *[4]*

8. Consider the power series $\displaystyle\sum_{n=0}^{\infty} x^{2n} = 1 + x^2 + x^4 + x^6 + \cdots$.

a. Determine the radius and interval of convergence of this power series. *[6]*

b. What function has this power series as its Taylor series? *[4]*

c. What power series is equal to the product

$$\left(\sum_{n=0}^{\infty} x^n\right)\left(\sum_{n=0}^{\infty} (-x)^n\right) = \left(1 + x + x^2 + x^3 + \cdots\right)\left(1 - x + x^2 - x^3 + \cdots\right) \text{ ? } [4]$$

[Total = 100]

Part D. Bonus problems! If you feel like it and have the time, do one or both of these.

3². Show that $\ln(\sec(x) - \tan(x)) = -\ln(\sec(x) + \tan(x))$. *[1]*

2 × 5. Write a haiku (or several :-) touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

Enjoy your summer!

P.S.: You can keep this question sheet. (Souvenir, paper airplane, fire starter, the possibilities are endless! :-) The solutions to this exam will be posted to the course archive page at `http://euclid.trentu.ca/math/sb/1120H/` *in late April or early May.*