**Mathematics 1120H – Calculus II: Integrals and Series**
Trent University, Winter 2022
**Final Examination**
*11:00-14:00 on Saturday, 23 April, in Wenjack.*

**Time:** *3 hours.* *Brought to you by* Стефан Біланюк.

**Instructions:** Do parts **X**, **Y**, and **Z**, and, if you wish, part **W**. Show all your work and justify all your answers. *If in doubt about something,* **ask!**

**Aids:** Open book, most any calculator, one head-mounted neural net.

**Part X.** Do all four (4) of **1**–**4**.

**1.** Evaluate any *four* (4) of the integrals **a**–**f**. *[20 = 4 × 5 each]*

**a.** $\displaystyle\int_{12}^{14} (x-13)^6\,dx$ **b.** $\displaystyle\int \frac{1}{z^2+3z+2}\,dz$ **c.** $\displaystyle\int_0^1 \frac{y\arctan(y)}{y+y^3}\,dy$

**d.** $\displaystyle\int u^3\sin\left(u^2\right)\,du$ **e.** $\displaystyle\int_0^\infty \frac{1}{(2v+3)^2}\,dv$ **f.** $\displaystyle\int \frac{2}{\sqrt{1+4w^2}}\,dw$

**2.** Determine whether the series converges in any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

**a.** $\displaystyle\sum_{n=0}^{\infty} 2^{-n^2}$ **b.** $\displaystyle\sum_{m=1}^{\infty} \frac{1}{\cos(m\pi)\cdot\sqrt{m}}$ **c.** $\displaystyle\sum_{i=0} \frac{i}{3^i}$

**d.** $\displaystyle\sum_{j=1}^{\infty} \frac{3^j}{j}$ **e.** $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{(k-1)!\cdot k^2}$ **f.** $\displaystyle\sum_{a=0}^{\infty} \frac{\sqrt{a}}{1+a^2}$

**3.** Do any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

**a.** Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{n}{17^n}x^n$.

**b.** Determine whether the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\,(n!)^2}{(2n)!}$ diverges, converges conditionally, or converges absolutely.

**c.** Find the volume of the solid obtained by revolving the region between $y = x-4$ and $y=0$, where $0 \le x \le 4$, about the $x$-axis.

**d.** Use the Left-Hand Rule to compute $\displaystyle\int_0^2 x\,dx$.

**e.** Find the sum of the series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$.

**f.** Find the area of the finite region between $y=x$ and $y=x^3$.

**4.** Consider the region between $y=\sin(x)$ and $y=0$, where $0 \le x \le \pi$. Solid A is obtained by revolving this region about the $x$-axis and solid B is obtained by revolving the region about the $y$-axis. Determine which of A and B has greater volume. *[12]*

**Part Y.** Do either *one* (1) of **5** or **6**. *[14]*

**5.** Consider the curve $y = x^2$, where $0 \leq x \leq 2$.

**a.** Find the area of the surface obtained by revolving the curve about the $y$-axis. *[7]*

**b.** Find the arc-length of the curve. *[7]*

**6.** A solid is obtained by revolving the region below $y = x^2$ and above $y = -\sqrt{4 - x^2}$, where $0 \leq x \leq 2$, about the $y$-axis. Sketch this solid and find its volume. *[14]*

**Part Z.** Do either *one* (1) of **7** or **8**. *[14]*

**7.** Recall that $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$. Find the Taylor series at 0 of $\cosh(x)$

**a.** using Taylor's formula, *[9]* and

**b.** without using Taylor's formula. *[5]*

**8.** Consider the power series $\displaystyle\sum_{n=0}^{\infty} \frac{n-2}{n!} x^n = -2 - x + \frac{x^3}{6} + \frac{x^4}{12} + \cdots$.

**a.** Find the radius and interval of convergence of this power series. *[8]*

**b.** Figure out what function has this power series as its Taylor series. *[6]*

*[Total = 100]*

**Part W.** Bonus problems! If you feel like it and have the time, do one or both of these.

**9.** Consider the following answers to a multiple-choice question:

*a.* The answer is *b*.
*b.* The answer is *c*.
*c.* The answer is *d*.
*d.* The answer is *e*.
*e.* None of the above.

Irrespective of the question, what should a student faced with this do? Explain! *[1]*

**10.** Write a haiku (or several :-) touching on calculus or mathematics in general. *[1]*

**What is a haiku?**

seventeen in three:
five and seven and five of
syllables in lines

ENJOY YOUR SUMMER!

*P.S.: You can keep this question sheet. (Paper airplane, fire starter, the possibilities are endless! :-) The solutions to this exam will be posted to the course archive page at* `http://euclid.trentu.ca/math/sb/1120H/` *in late April or early May.*