**Mathematics 1120H – Calculus II: Integrals and Series**
TRENT UNIVERSITY, Winter 2022

**Assignment #9**
**Power Series**
*Due on Friday, 25 March.*
*(May be submitted on paper or via Blackboard.*)*

Please show all your work. As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

Please read §11.4 in the textbook and review the lecture on the Alternating Series Test and conditional *vs.* absolute convergence before tackling tis assignment.

**1.** Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\ln(n)}{n} = \frac{\ln(1)}{1} - \frac{\ln(2)}{2} + \frac{\ln(3)}{3} - \frac{\ln(4)}{4} + \cdots$ converges conditionally, converges absolutely, or diverges. *[2]*

A *power series* is a series of the form $\displaystyle\sum_{n=0}^{\infty} a_n x^n$, where $x$ is a variable and each $a_n$ is a constant. It's basically a polynomial of infinite degree, one major difference being that while a polynomial of finite degree will be defined for all $x$, there is no guarantee that a power series will converge for all $x$. Indeed, there is no guarantee that it will converge for any $x$ other than 0:

**2.** Show that the power series $\displaystyle\sum_{n=0}^{\infty} n!x^n$ converges only when $x = 0$. *[1]*

NOTE: If $k$ is a positive integer, then $k!$ ("$k$ factorial") denotes the product of all the positive integers less than or equal to $k$, *i.e.* $k! = k(k-1)(k-2)\cdots 3 \cdot 2 \cdot 1$. To make all sorts of formulas work nicely when $k = 0$, we arbitrarily define $0! = 1$.

HINT: $k!$ grows very fast with $k$, eventually overtaking and then running away from $a^k$ for any $a \in \mathbb{R}$. You may use this fact without further ado.

**3.** Express $\dfrac{1}{1+x}$ as a power series. For which values of $x$ does the series converge? *[2]*

**4.** Express $\ln(1+x)$ as a power series. For which values of $x$ does it converge? *[2]*

HINT: How is $\ln(1+x)$ related to $\dfrac{1}{1+x}$?

**5.** Use your answer to **4** to find the sum of the alternating harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$, then check your answer by using `SageMath`. *[2]*

* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`

Hint: SageMath has a sum command, which we saw way back in Assignment #1. It can be used to sum infinite series, too.

**6.** Use SageMath to find the sums of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$. *[1]*


(This comic appeared on 2022-03-13.)